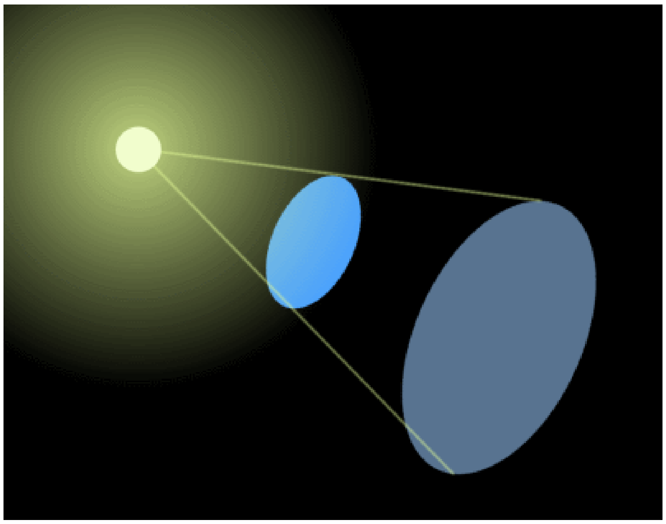
The Mother of all Averages

By Alan Siddons, May 2009

A blackbody "sphere" really consists of a flat disk that's been expanded 4 times and placed at twice the distance from the sun, thus allowing the inverse square law to reduce radiance on that disk by four times as well. 4 times larger, but also 4 times less energized. Given 1 unit of irradiance on a disk, then, the same irradiance on a blackbody sphere equals 0.25 units.

The question is, does this model mimic reality?



Here's the justification given by computer modelers.

A disk's surface area is defined as its radius squared × pi. So, with a radius of 1, a disk will have a surface area of pi, 3.14, whilst a sphere's area is defined as its radius squared × pi × ***4***. The same disk converted to a sphere will therefore have a surface area of 12.57, four times more than the original disk. Now, because this converted disk is four times larger but exposed to the same amount of energy, each part receives four times less energy. It's the same effect as diluting dye in water.

To labor the obvious even further, a sphere consists of two hemispheres, call them A and B. When a light source illuminates A, it leaves B in the dark. Since each hemisphere has twice the surface area of a disk, X watts per square meter directed at Hemisphere A therefore entails ***X divided by 2*** W/m^2 spread over its surface and of course ***0*** W/m^2 spread over Hemisphere B. The average amount of light absorbed by A and B together, then, is $(0.5X + 0) \div 2 = 0.25$ X.

In short, it always works out the same: a sphere absorbs four times less per surface area than a disk. Thus it seems logical to calculate temperature on this basis.

There's a problem, however. A huge one.

Let's do some calculations with real numbers.

- A blackbody disk exposed to 100 W/m^2 reaches a temperature of 205 Kelvin.
- Under the same circumstances a sphere supposedly absorbs four times less and reaches 145.
- But two hemispheres will reach 172 and 0 respectively, thus averaging 86 Kelvin, or 59% of the temperature predicted for a sphere.

To clarify this, consider that if a planet keeps one face to the sun, half the planet's surface area is constantly absorbing half of the available radiance while the other half absorbs nothing. Just as a radiative constant applies directly to a disk, half of that constant applies to a hemisphere. As stated above, the result is 0.5 × radiance and 0 × radiance, yielding two temperatures to average, 172 and 0 Kelvin in this case.

In fact, in order to reach the assumed average temperature of 145 in this case, the light-receiving hemisphere must be exposed to ***800 W/m^2***, exactly eight times more radiant energy!

Thus, for a planet keeping one face to the sun, the traditional divide-by-four formula for temperature is completely unjustified. The method preemptively robs Peter to pay Paul, underestimating the illuminated hemisphere's temperature in order to add heat to the other one.

Yet at any instant of time, ***every*** planet has only one face to the sun. Instant by instant, one hemisphere is absorbing all the radiant energy available while the other is absorbing none. No matter how you try to change the scenario, you cannot alter the fact that one side is lit and the other side dark. This is a fatal error in conventional blackbody temperature estimates. I fear it cannot be corrected.

In short, the hemispherical formula $(0.5X + 0) \div 2 = 0.25$ is a perfectly valid description of average radiance absorbed on a complete sphere. But this formula must be adhered to for temperature as well, although the result is quite different from what people have come to expect.

As proof of this method's illegitimacy, notice that if you follow the divide-by-four formula, you cannot answer the simple question of how warm an illuminated hemisphere is. You have only an average spherical temperature to go by with no idea of what figures compose the average. That is another fatal flaw.

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Perhaps the first thing to point out about the geometrically justifiable rule of (Kelvin + 0) ÷ 2 is that it is most applicable to a massive sphere whose conductivity can be regarded as 0.

To understand this in converse terms, take a round black pebble.

Exposed to 100 W/m², the pebble's outer surface will initially transfer warmth to its interior. In other words, the pebble will take time to warm up. Its temperature change won't be immediate. Once conductive transfer has gone as far as it can, there's no other means to store the heat, and so the surface temperature will climb to a maximum, averaging 172 on the hemisphere facing the radiance.

But what of the other hemisphere? If the pebble is small enough, it's conceivable that 100% of the pebble's heat can migrate to the cold side, in which case both sides of the pebble will be at 172 Kelvin, ***an average temperature 19% higher than predicted*** for a sphere absorbing 25% of the available radiance.

(Note: the blackbody limit here is 205 Kelvin. Geometry is one factor but conductivity is another. This example doesn't contradict the rules of radiative forcing or heat transfer.)

The larger the object, the less can conductive transfer add to the cold side, but there's still its stored heat to consider. If the sphere in question is a rotating planet and its soil holds onto 20 degrees during the night, then the two sides will average (172 + 20) ÷ 2, i.e., 96 Kelvin. The planet will be "hotter" than predicted but due to nothing more than a surface possessing depth and not releasing its heat instantaneously.

Dividing a sphere's radiant energy by four is geometrically unjustified, then, a wild stab in the dark. Unless one knows how much heat the sphere retains during rotation, there is no way to estimate its average temperature. A blackbody calculation is merely guesswork that an actual physical body is under no obligation to obey. Qua sphere, a body can reach a temperature of (K + 0)/2 all the way up to (K + K)/2, temperatures lower and higher than a simplistic formula that divides by four.

As a corollary, these facts also demonstrate that there's no such thing as "radiative equilibrium," i.e., no condition set by a blackbody calculation that forces a planet to adjust its temperature.

It is believed, for instance, that since earth's "true" radiant absorption is 240 W/m², then anything which limits an equivalent emission must be compensated for by raising the temperature until that barrier is broken and emission is equal to absorption. Thus a radiative bottleneck is presumed to compromise the earth's emission such that an extra 150 W/m² are required to emit 240 W/m² in total. By this logic, a 288 Kelvin surface emits 390 W/m² that get bottlenecked – but since 240 emerge, the 240 W/m² criterion is satisfied.

Yet nothing defines this criterion except a loosely formulated generalization that doesn't conform to reality. A legitimate temperature estimate must begin by assuming half-lit and half-dark and proceed from there.

In summary, both the radiant energy absorbed and a planet's consequent temperature can only be guessed within a range of mathematically reasonable possibilities, beyond which actual empirical measurements are demanded.

There is no reality in a divide-by-four radiance/temperature formula and thus no physical basis for "radiative equilibrium."

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