# Does Surface Area Determine Temperature?
by Alan Siddons

Anyone keeping pace with the global warming debate has undoubtedly seen the claim: The Earth is 33 degrees warmer due to the extra thermal radiation that Greenhouse Gases provide. Acceptance of this basic claim is so ingrained that it's nearly an article of faith. But it does have a rationale behind it, which we'll examine here. Briefly, the argument runs as follows.
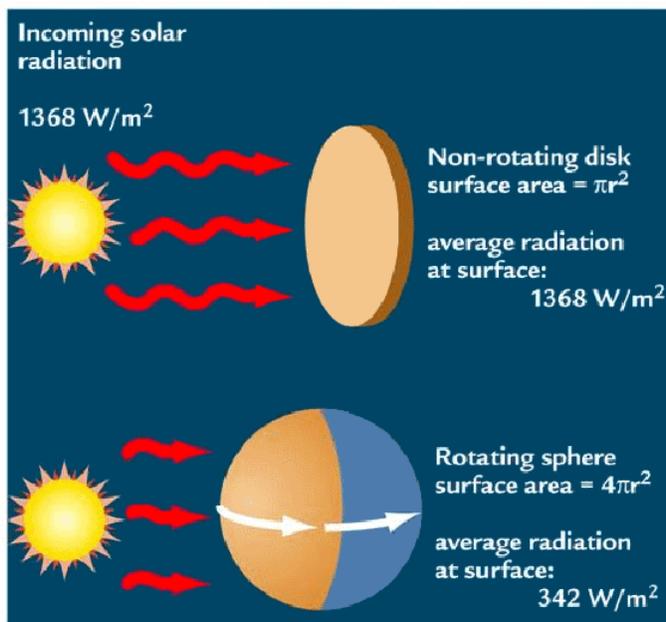

If the Earth were a flat disk facing the Sun at a 90° angle, it would be exposed to the maximum intensity of solar radiation. But the Earth is spherical, and a sphere of the same diameter has four times the surface area as a disk. Sunlight distributed over this greater area thus has a heating impact of four times less.

While the Sun is able to impart about 1368 watts per square meter to a disk, then, the same light doled out to our spherical planet has an effective power of only 342, or ¼ the strength. Taking 30% reflectiveness into account, this heating potential is further reduced to about 239 W/m², which translates to 255 Kelvin for a theoretical entity called a 'black body.'

This is why it is claimed that the Earth is 33 degrees warmer than explainable by sunlight alone, since 288K is closer to its actual average temperature.

To recap, then, the argument states that light diffused over a sphere encounters four times more surface area than a sun-facing disk offers, and this reduces light's heating effect to 25%.

Given that a sphere has four times the area, however, it follows that a hemisphere has two times, and should therefore absorb *half* as much energy as a flat surface. Indeed, assuming that the sun-facing side of a globe gathers 50% of the available light while the other side is gathering nothing, their net contribution would be 25% (50% divided by 2), just as the surface area argument says. Let's focus on the sun-facing side of a globe, then, and see what goes on there.
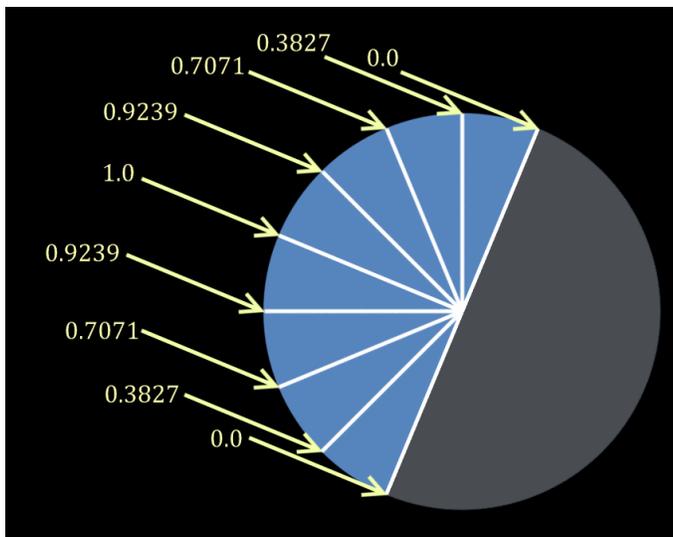
It's immediately apparent that a round surface simultaneously presents many angles of incidence to sunlight, unlike a flat surface. In the diagram here we've assigned an intensity of 1 to light that falls vertically, the same intensity that a sun-facing disk enjoys. In other words, at this single location the Sun is at 90°, or directly overhead, and has a peak thermal impact. From there, light intensities along this curving body follow a sine function. Let's explain what that means.

Say a flashlight beam is bearing down on the 90° mark of a giant protractor. The narrow beam in this position spans 6 degrees. So we'll use 6 as the standard for peak strength. The same beam centered near the 37° mark spans about 10 degrees, which naturally reduces its intensity.

Notice this then: 6 divided by 10 is 0.6, which suggests that the beam at 37° angle has about 60% of the peak's intensity. And what is the *sine* of 37°? 0.6018. Our guesstimate was pretty close. By the same token, the beam
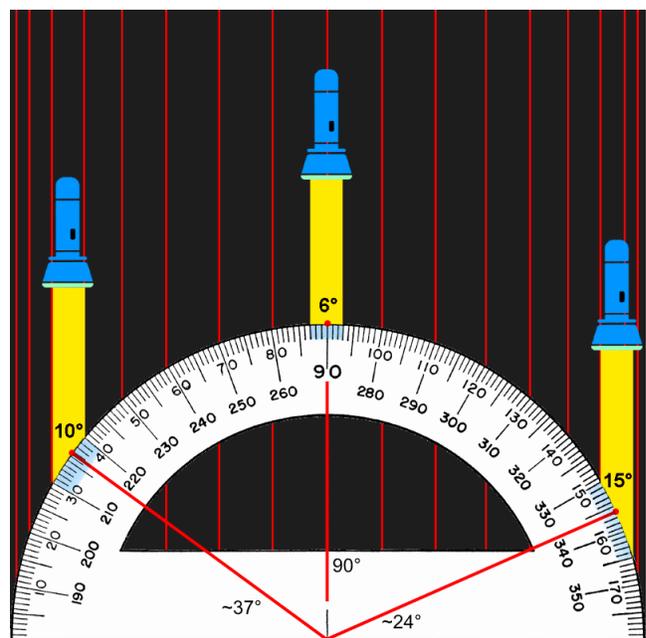
centered near 24° spans around 15 degrees. 6 divided by 15 is 0.4 – and the sine of 24° is 0.4067. This should give you a rough idea of how sines pertain to irradiance. At bottom, a sine is merely one numerical value compared to another.

The sine of 90° is 1, signifying maximum intensity. As a light beam's angle of incidence gets closer to 0°, the irradiance gets weaker since it follows a sine function. The question this essay addresses is, *what is the average sine (i.e., irradiance) on a semicircle that's facing a light source?* Our 3 samples together average 0.6695, but it's clear that we should use more samples.

Before we proceed, though, one more point: It should go without saying that the irradiance

The average irradiance of sunlight along a semicircle's curved edge is the same at any angle it rotates.
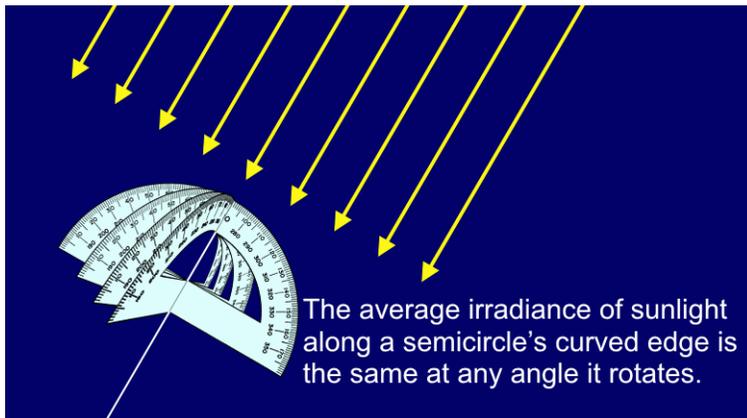
on a semicircle pointed at the Sun will remain the same no matter what angle it assumes on its axis. R**otating a semicircle merely allows it to catch the light rays that would be absent at another angle.** This means that a semicircle's average irradiance equally represents the irradiance on a 3 dimensional hemisphere, a subject we shall revisit.
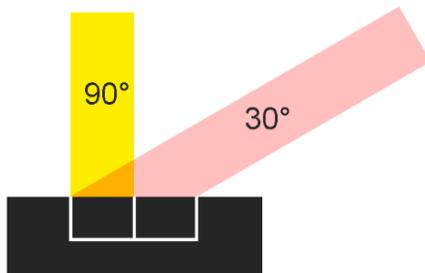
At a 30° angle a light beam is spread out twice as much and has half the strength. This value, 0.5, is also the sine of 30°

Up to now we've seen that the surface area argument implies that an illuminated hemisphere's average sine is 0.5, because its average irradiance is 50% compared to a sun-facing disk.

Let's cut to the chase, then. Since irradiance angles on a semicircle are the same from zero to 90° as they are from 90° to zero, averaging the sines from 90° to zero suffices. And in 1 degree increments from zero to 90° the average sine works out to be 0.6351, which is greater than 0.5. If you incorporate the sines for *thousands* of fractional angles between zero and 90°, though, you'll see average irradiance close in on a limit of around 0.6366. Why should this be so?

Well, think back to that sun-facing disk.

The circumference of that disk (blue circle) is its diameter times pi. Along every point on its surface, that circle is exposed to the maximum radiant power, which we've set as 1.

sunlight = 1
diameter = 1
circumference = π

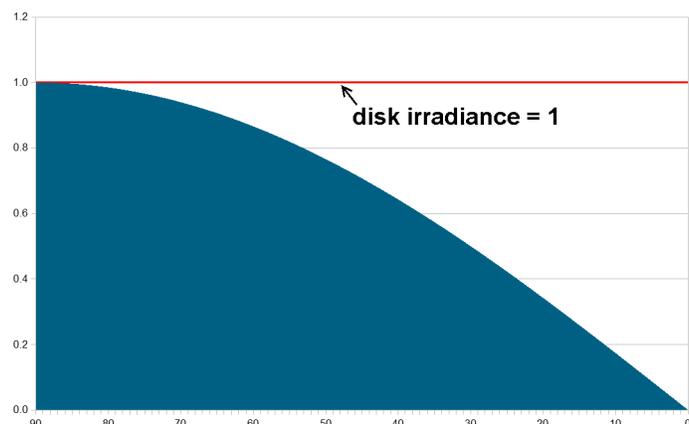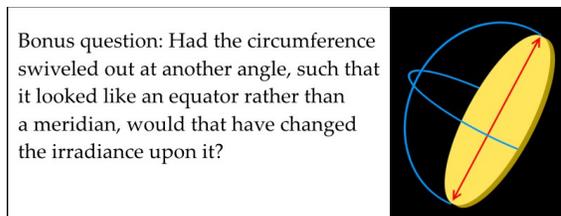meridian = pi/2
diameter = 1
circumference = π
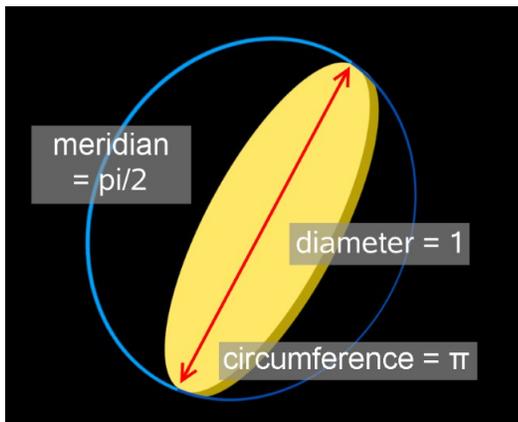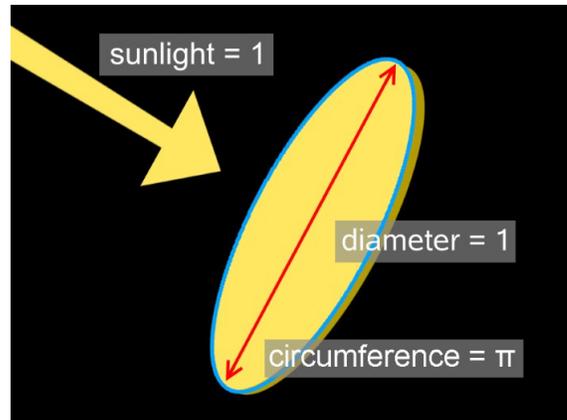
But if we rotate that circumference to face the Sun on edge, the irradiance on it changes radically. The Sun hits only one point on the circle directly, and half of it sees no Sun at all. This irradiated semicircle, then, has a length of pi/2, which is approximately 1.57.

Now as noted above, the more angles we account for on an illuminated meridian, the more the average sine approaches 0.6366. Here's the reason: Given an intensity of 1 for sunlight falling directly on a disk, the same intensity spread across a 1.57 meridian amounts to **1 divided by half of pi** – i.e., 0.6366. This is the figure that successive trigonometric measurements keep pointing at.

While it's incontestable that the surface area of a hemisphere is twice that of a disk's, then, it remains that a light beam's intensity follows a sine function, which makes it impossible for an irradiated meridian (or hemisphere) to see less than 63.66% of the light impinging on it. Here is what the profile of such an irradiated meridian looks like, starting from a 90° angle of incidence.

Please notice that for a curving surface that is continuously turning toward a beam of light, a maximum intensity of 1 can only occur at 90° of incidence, the same as for a sun-facing disk.

Bonus question: Had the circumference swiveled out at another angle, such that it looked like an equator rather than a meridian, would that have changed the irradiance upon it?


disk irradiance = 1

In that regard, if it's *true* that an irradiated hemisphere acquires only a 50% irradiance compared to a disk, then we ought to ask what happens at 90° of incidence. If the value *does* match the disk's, then here is the irradiance profile that necessarily results.
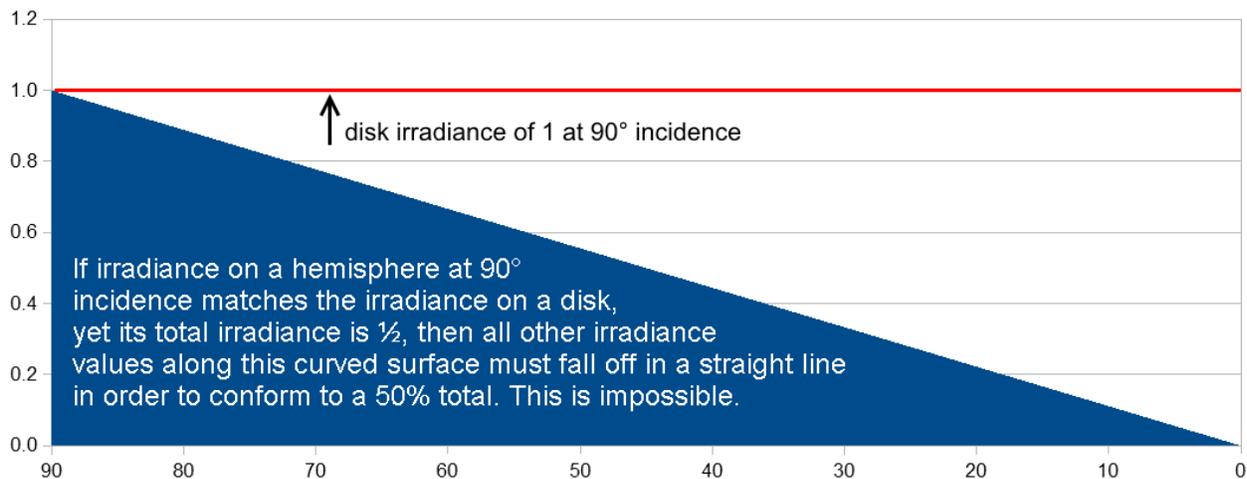

Notice now that we *can* avoid the embarrassment of a straight-line irradiance on a curved surface, but only at the price of ignoring the fact that the two irradiances MUST match at a 90° incidence. While the profile below *does* conform to a round surface continuously turning toward a beam of light, then, it also conforms to a 50% irradiance, thereby falling short of equality by 21%.
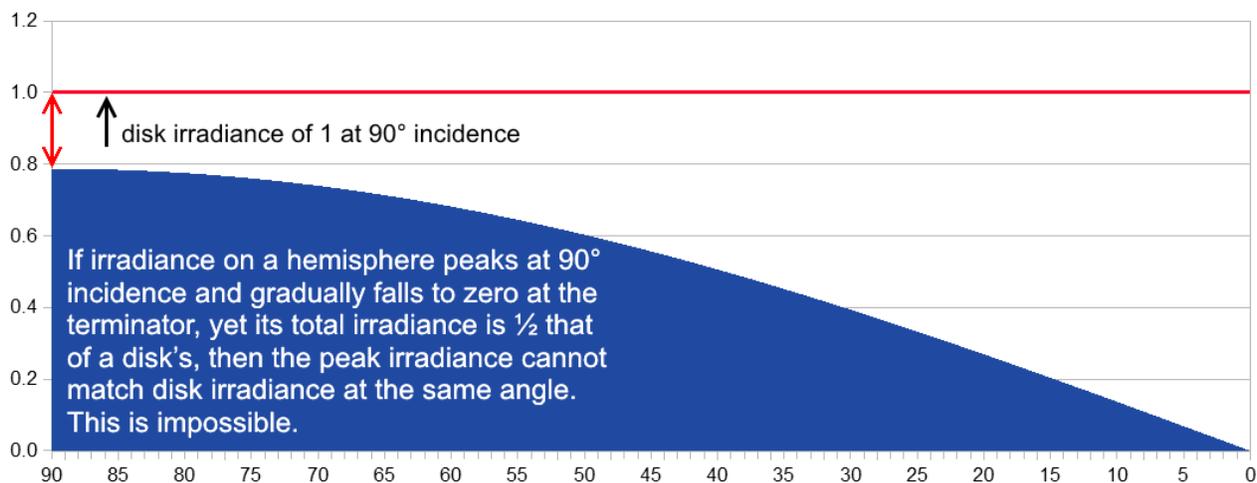
In sum, the two scenarios for the profile of a hemisphere with a 50% irradiance are both impossible. By contrast, a 63.66% scenario fits the necessity for equal irradiance at 90° **and** for the way light intensity changes on a round surface.
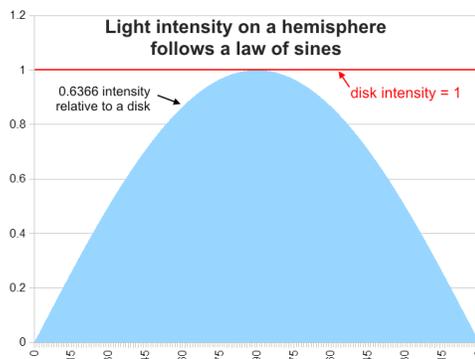
Still not convinced, though, that a meridian line's irradiance stands for a hemisphere too? Well, I can only repeat what I said above, that twirling a protractor while it's aimed directly at the Sun does nothing to alter the irradiance along its circular edge. An illuminated meridian's 0.6366 average stands for *any and all such lines*. Here, for instance, the green line will report the same radiant average as the red one. So would a third line, a thousand lines, a million.... till they constituted an entire hemisphere.
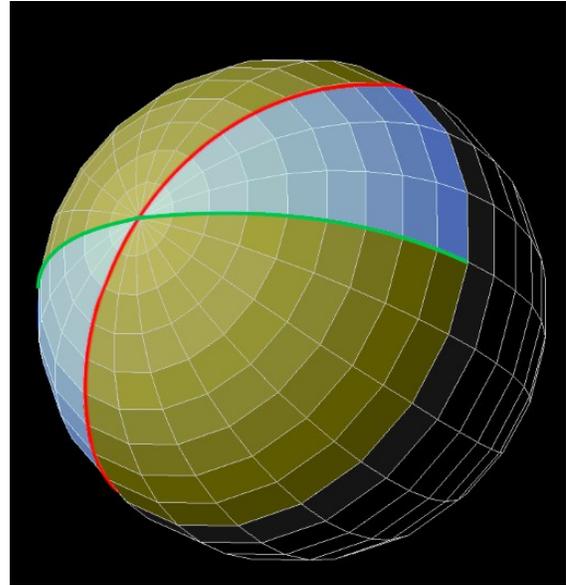

A beam of light doesn't discriminate; it always falls on a hemisphere or a selected meridian the same way. The average power of sunlight on a hemisphere or on any number of exposed meridians is 0.6366 by necessity.

One can understand this from yet another perspective, however, for a beam of light creates a ripple of irradiance rings on a hemisphere, circles of illumination which each have a particular intensity. The intensities of thousands of such rings will also average 0.6366.

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There's apparently no way around it: A hemisphere is able to intercept nearly 64% of available solar energy, not 50%. Since the other hemisphere is entirely cut off from sunlight, this gain is split in half, making the irradiance average over a whole sphere conform to the simple **inverse of pi**, i.e., 1/pi, or 0.3183. That's roughly 32%. But definitely not 25%.
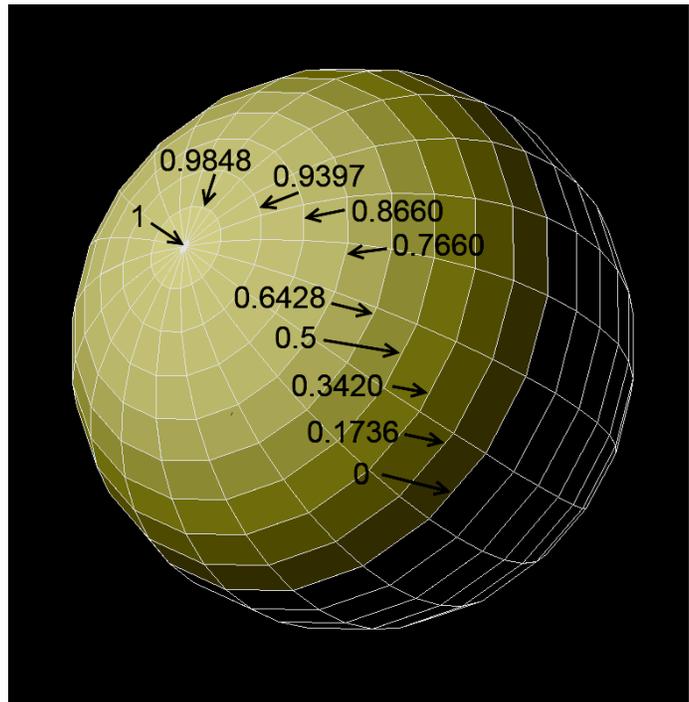
In short, the surface area argument is geometrically faulty. Angle of incidence determines the radiant power striking a sphere, not its surface area compared to a flat disk, and as a result a sphere is able to absorb more light than figured by the area assumption.

This essay's sole focus was to test a single tenet of radiative greenhouse theory, not delve into details like the unusual heating profile of a hemisphere absorbing 63.66% of the available light. Nevertheless, let's touch on temperature briefly, using the same (overly) simple arithmetic. Earlier we mentioned that the real Earth absorbs about 70% of sunlight's energy and reflects the rest. Including the long-held 0.25 distribution factor thereby produces this prediction:

$$1368 \text{ W/m}^2 \times 0.7 \times 0.25 = 239.4 \text{ W/m}^2$$

– which translates to 255 Kelvin. This differs from 288 K by 33 degrees. A distribution factor of 0.3183, on the other hand –

$$1368 \text{ W/m}^2 \times 0.7 \times 0.3183 = 304.8 \text{ W/m}^2$$

– translates to 271 Kelvin. The difference between this and the actual average temperature is around 17 degrees rather than 33, which slices the purported 33° discrepancy nearly in half.

Quite naturally this should trouble anybody who's trusted the basic tenets of greenhouse theory, for it not only undermines the theory's *very first premise* but invalidates the compensatory 'radiative forcing' magnitudes that this premise engendered.

Here's a quick review of this irradiance issue along more intuitive lines. The European Space Agency lists 1/pi as the average sunlight absorbed by a cylindrical satellite in space, which makes perfect sense. Take a ring, assign the perpendicular irradiance on it a value of 1, and spread (distribute) that irradiance over the entire circumference, pi. This yields 1 divided by pi, of course.

Beam of light illuminating a ring
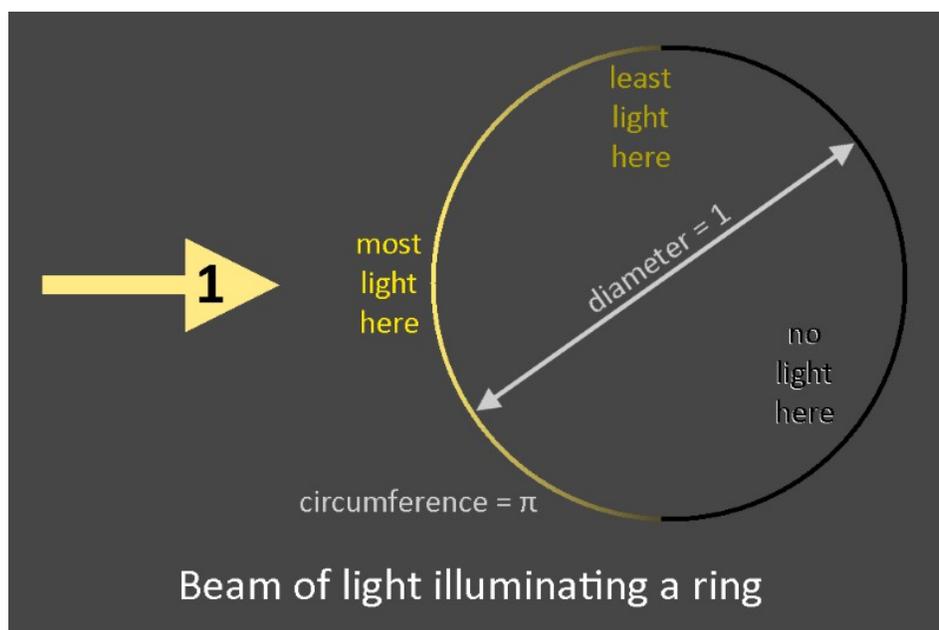
1 divided by pi most definitely applies to an irradiated cylinder, then, because the shape of a cylinder is merely a series of such rings.

net light absorption = 1/π
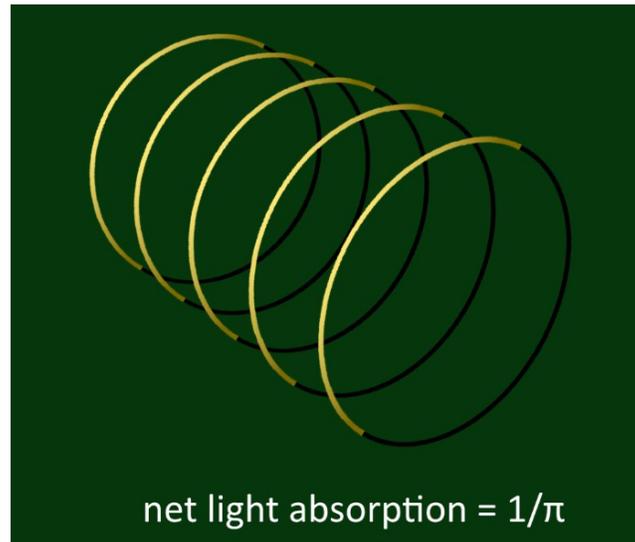
By the same token, though, the shape of an irradiated sphere is also a series of such rings.

net light absorption = 1/π
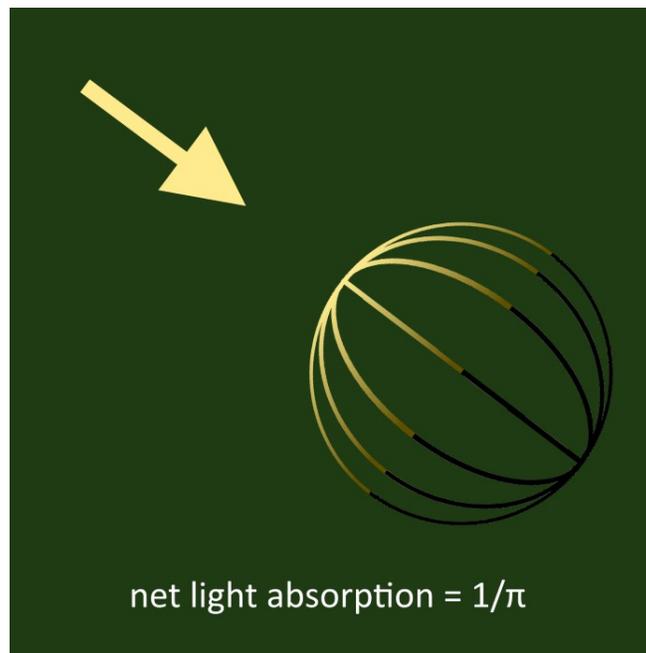
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(revised April 2018)