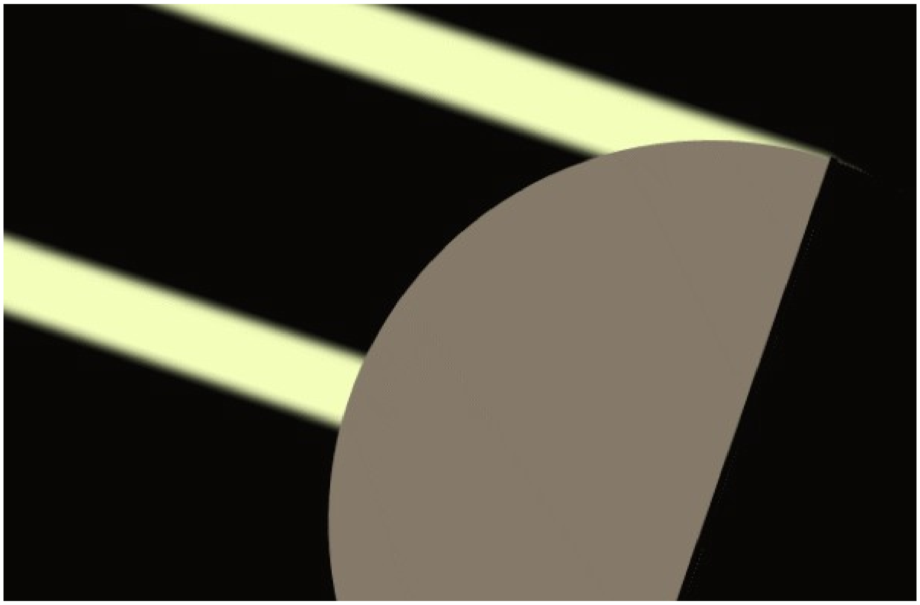
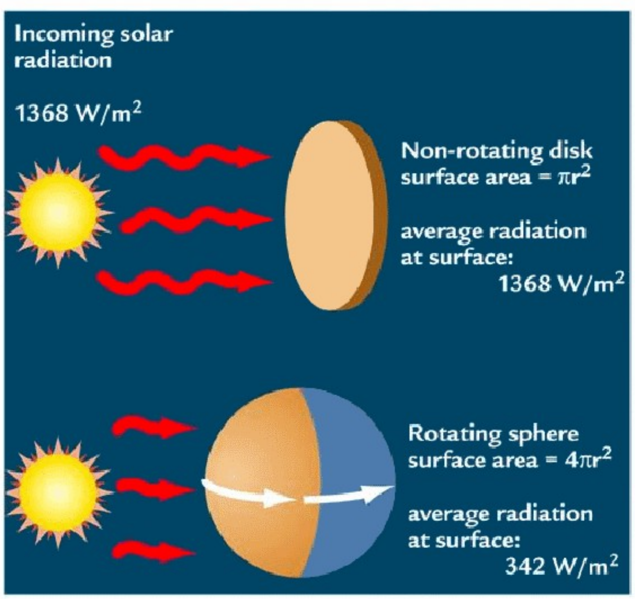


Does Surface Area determine Temperature?

Anyone keeping pace with the global warming controversy has undoubtedly seen the claim: The Earth is 33 degrees warmer due to the extra thermal radiation that so-called Greenhouse Gases provide. Acceptance of this claim is so ingrained by now that it's nearly an article of faith. But it does have a rationale behind it, which we'll examine here. Briefly, the argument runs as follows.



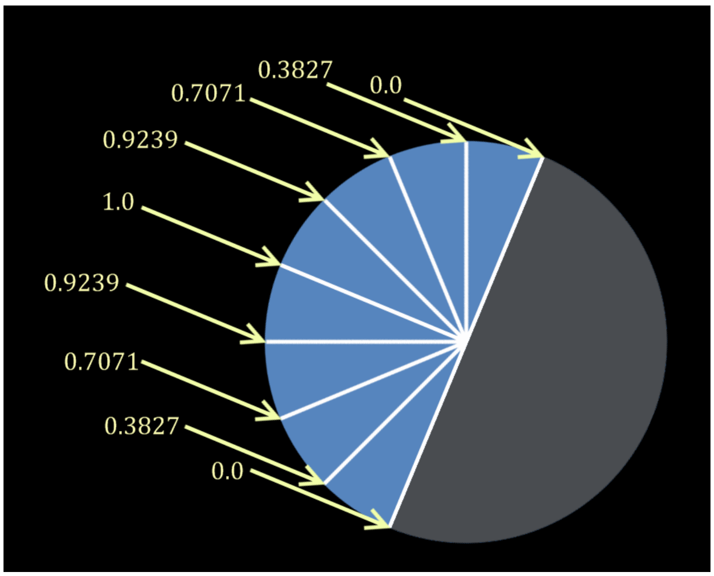
If the Earth were a flat disk directly facing the sun, it would be exposed to a maximum intensity of solar radiation. The Earth is a sphere, though, and a sphere of the same diameter has four times the surface area as a disk. One must therefore conclude that sunlight distributed over this greater area has a heating impact four times less.

Thus, while the sun is able to impart about 1368 watts per square meter to a disk at the Earth's distance, the same light distributed over a spherical body has an effective power of only 342, or ¼ the strength. Taking the Earth's 30% reflectiveness (non-absorption) into account, this heating potential is further reduced to about 239 W/m², which translates to minus 18° Celsius for a theoretical entity called a 'black body.' This is the basis of the claim that the Earth is 33 degrees warmer than explainable by sunlight alone, since +15° is closer to its true average temperature.

To recap, the argument states that light spread over a sphere encounters four times more surface area than a disk presents, and this reduces sunlight's heating effect to 25%. Given that a sphere has four times the area, though, it's likewise true that a hemisphere has two times.

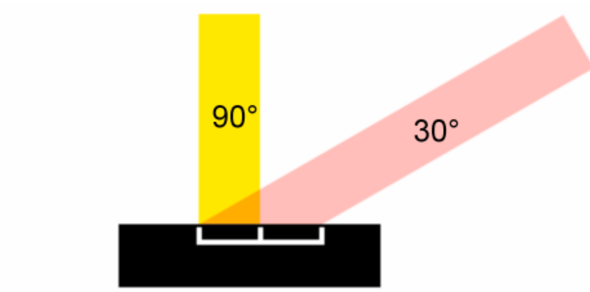
And a hemisphere is actually the best place for us to start, it turns out, because in the real world sunlight only falls on half our planet at a time, not over all the Earth at once. So if the sun-facing side of a globe gathers 50% of the available light while the other side gathers zero, then their net contribution will be 25%, just as the surface area argument indicates.

Let's focus on the sun-facing half of a globe, then, and see what we find.



It's immediately apparent that a round surface offers many angles of incidence to sunlight, unlike a flat disk. In this diagram we've assigned an intensity of 1 to light that falls vertically on the surface. In other words, at this single location the sun is at 90°, or directly overhead.

From there, the intensity of light along this arc follows a sine function. For example notice that 45 degrees away from vertical — i.e., halfway between the 1 and zero intensities — radiant power hasn't fallen to 0.5, as one might guess, but merely to 0.7071, the sine of 45°. Indeed, light only reaches half-intensity at a 30 degree angle, fully 60 degrees away from perpendicular.

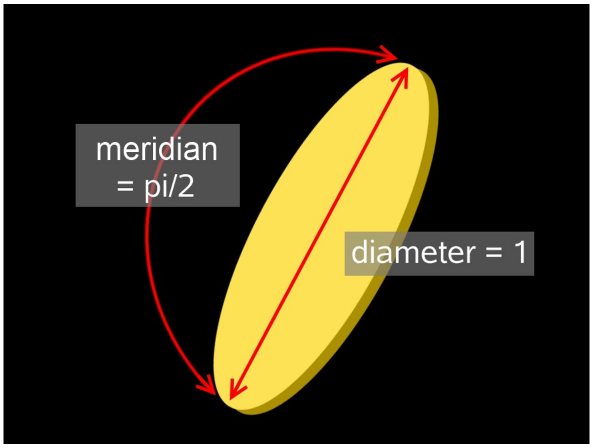


At a 30° angle a light beam is spread out twice as much and has half the strength. This value, 0.5, is also the sine of 30°

So here's the crucial question: What is the **average sine value** for light's multiple angles on a hemisphere? That value will represent light's average intensity on its surface.

Now, so far we've seen that the surface area argument requires a sunlit hemisphere to have an average value of 0.5 — as if light is impinging at a 30° angle on a flat surface — because radiant energy has been spread out over twice the area. Yet this means we've already hit a snag. For although the nine points in our sine illustration are rather widely spaced (at 22½ degrees apart), the light intensity they record averages 0.5586. This obviously exceeds 0.5, the value we're looking for. But will the average of more sines resolve this disagreement? No, it will not.

When you record the sines at points spaced 10 degrees apart, for instance, the irradiance average grows larger: 0.6016. Using 5 degree increments the average sine rises to 0.6190. Curiously, then, more precision brings about more disagreement with a predicted 0.5 illumination, not less. If you persist, though, and incorporate sines for the many, many fractional degrees in a half-circle, the average irradiance value chokes off at around 0.64. **Why should this be so?**



Well, think back to that sun-facing disk. Call its diameter 1. A sphere with the same diameter will therefore have a circumference of 1 × π, or 3.14. But the sun is only able to light half a sphere at a time. So a meridian line drawn across this illuminated half will have a length of ½ **π**, or roughly 1.57.

Now, as discussed above, a half-circle intercepts a beam of light at angles ranging from 90 to 0 degrees. And we've seen that the more angles we account for the more the average sine approximates 0.64. But why? Here's the reason: given an intensity of 1 in direct sunlight, that brightness value distributed along a 1.57 meridian line amounts to **1 divided by half of π** — i.e., 0.6366. This pertains to another unit of measurement, the radian, a subject we won't pursue here. Suffice it to say that 0.6366 represents the irradiance average that successive trigonometric measurements will approach.

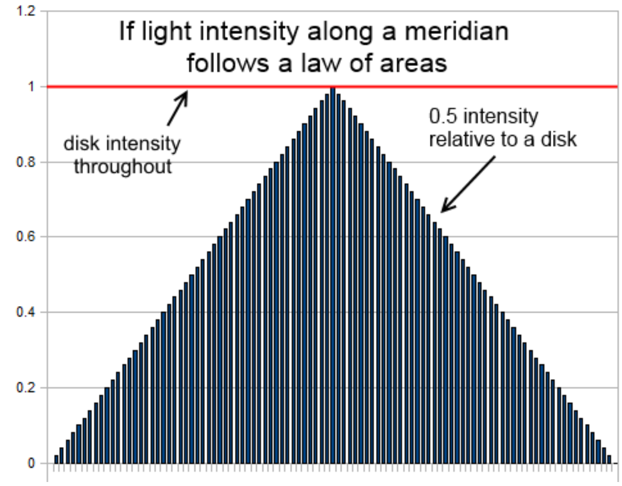
So what is this hinting at? Well, sure, it's incontestable that the surface area of a hemisphere is twice that of a disk's. But the intensity of sunlight on a hemisphere or on any surface nevertheless follows a sine law, which makes it impossible for a sunlit meridian line to see less than 63.66% of the radiant energy beaming on it. An intensity reduction to 50% just isn't in the cards.

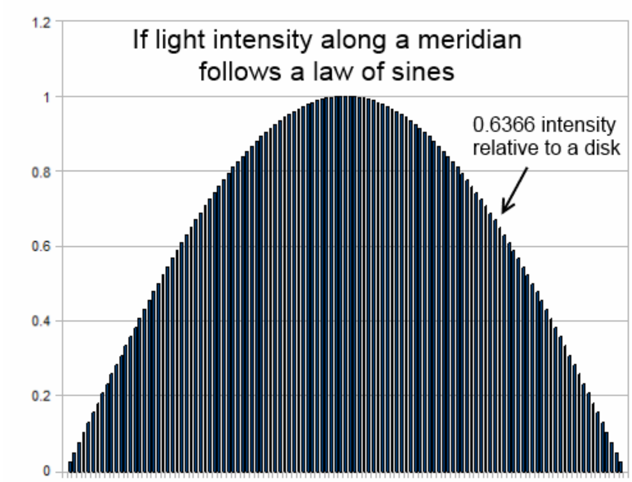
All the same, let's imagine for a moment that light's intensity on a hemisphere does not conform to every angle of incidence it encounters but somehow adjusts itself to the area it covers compared to a disk. In this alternate reality sunlight trims its average power to ½ by adapting only to the surface area. Since a sun-facing disk sees an intensity of 1, then, a hemisphere will see ½ *in toto*.

Do we grant, though, that a 90 degree incidence angle is equal to 1? Then that's the intensity for a 90 degree incidence. Do we also concede that at 0 degrees incidence the irradiance is zero? Then it's simple. Every irradiance value between 1 and zero must proceed linearly in order to obtain an average of 0.5. No way around it.

So here we start from a 0 degree angle of incidence, follow the meridian in uniform steps till we reach 1, and then proceed to the other zero on the line. We've recorded the surface's light intensity with every step, and here is the record.

Whereas the average light intensity on a disk is represented by that horizontal red line drawn across the value 1, the light on a hemisphere forced us to obtain an average of 0.5. And the outcome is laughable. It goes without saying that such a misguided scenario will always make a hemisphere's interception of light look triangular no matter how many steps are chosen.





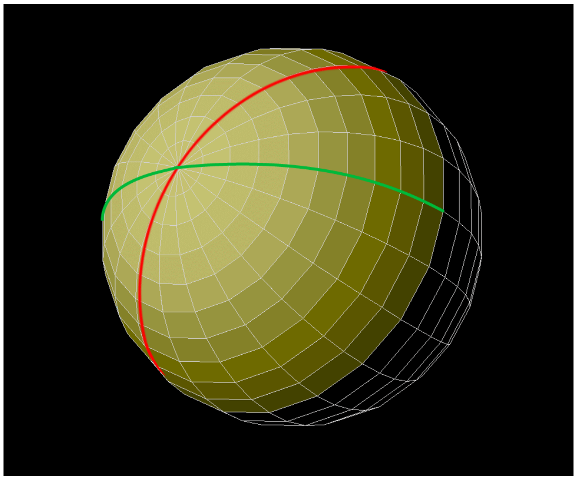
By contrast, observe the pattern for a meridian march with each sine value inserted.

This step-by-step record of radiant intensities attests to an average of 0.6366, appreciably greater than what a surface area argument demands.

Well, enough of alternate realities. The angle of incidence certainly does determine a light beam's heating ability and it obeys a trigonometric rule. In retrospect, perhaps, we can appreciate how difficult it is to avoid the number pi when we're dealing with a sphere.

A point to bear in mind, however, is that a single meridian's 0.6366 average stands for any and all such meridians.

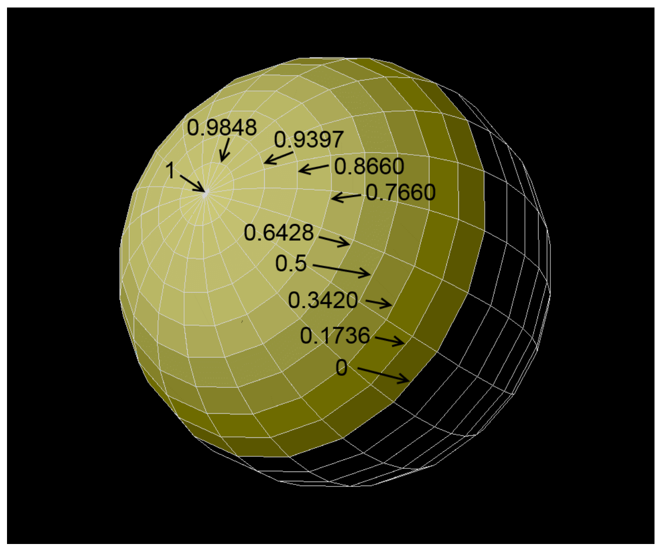
Light doesn't care about cartographic latitude and longitude or what may be a north or south pole. It simply spreads out on a surface according to its angle or angles of incidence.



Here, for instance, the green line will report the same average light intensity as the red line. So would a third line and a fourth. Any randomly drawn radiant meridian will have the same value. The average light intensity for a single line or a group of them will always be 0.6366.

For indeed, a hemisphere is basically composed of an infinite number of intersecting meridians. This means that if we were to manually construct a hemisphere, meticulously building it with billions of slender arcs — each one of which catches about 64% of the light from an overhead sun — 64% will likewise represent what the hemisphere itself is exposed to.

One can understand this from yet another conceptual vantage point, however, for a beam of sunlight also creates what amounts to an infinite series of irradiance rings on a hemisphere, not just across it. These circles of uniform illumination each have a specific intensity value. And guess what? A multitude of these rings also averages 1 divided by half of pi, i.e., **0.6366**. Across the full span of a hemisphere or covering it with concentric rings, light on the surface of a globe keeps reverberating π.



Bottom line, a hemisphere is able to intercept nearly 64% of an incident light beam, not 50%. Isolated from any light, of course, the other hemisphere necessarily cuts this gain in half, making the irradiance average for a whole sphere conform to the **simple inverse of pi: 1 ÷ 3.1416 = 0.3183**.

That's roughly 0.32, or 32%. But not 25%.

To sum up, the surface area argument is geometrically faulty. Angle of incidence determines irradiance on a sphere, not more surface area relative to a flat plane, and as a result a sphere is able to absorb more light than a surface area assumption predicts.

Let's redo the math, then. Earlier we mentioned that the Earth's components absorb about 70% of sunlight's energy and reflect the rest. So the inclusion of a 0.25 distribution factor produces this result:

$$1368 \text{ W/m}^2 \times 0.7 \times 0.25 = 239.4 \text{ W/m}^2$$

— which translates to 255 Kelvin or minus 18.25° Celsius by the Stefan-Boltzmann equation. Minus 18° is what climate authorities have claimed would be the Earth's average temperature without greenhouse gases. Given an average irradiance of 0.3183, however, you get

$$1368 \text{ W/m}^2 \times 0.7 \times 0.3183 = 304.8 \text{ W/m}^2$$

— which translates to 271 Kelvin or minus 2.38° C. The difference between this (merely arithmetical figure) and the Earth's actual temperature is thus around 17 degrees rather than 33, which slices the Area Argument's assumption nearly in half.

These considerations should of course trouble anyone who's trusted the basic tenets of greenhouse theory, because not only do they undermine the theory's *very first* premise, they skew the numerous 'radiative forcing' calculations that this premise has spawned. The theory of heating by 'greenhouse gases' has never been very scientifically rigorous, though, so it's doubtful whether this essay will sway many of its adherents.

Alan Siddons