

Radiation Physics Laws Give the Effect of CO₂ on Earth's Temperatures – A Primer

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Abstract

A new chemical process control systems engineering model of Earth's atmosphere quantifies the effect of CO₂ on Earth's surface temperature. It uses the rigorous S-B radiant energy transfer rate law. The Earth's surface and atmospheric temperatures are given explicitly as linear ordinary differential and algebraic equations; the only system properties needed are absorptivity and emissivity, five of which depend on CO₂.

CO₂ affects surface temperature by at least four mechanisms, one positive and three negative. CO₂ decreases Earth's global radiating temperature to space slightly. When atmosphere parameters increase by 1% due to CO₂, surface temperature changes - 0.762C and atmosphere changes - 0.392C.

1. INTRODUCTION

Astrophysicists and climatologists have been struggling to quantify the effect of atmospheric CO₂ on Earth's temperature since the 1997 Kyoto Protocol called for control of Earth's global temperature by human throttling of hydrocarbon combustion, which emits CO₂ to the atmosphere. Kyoto called for building a thermostat for the whole Earth. The greenhouse gas theory, GHGT, is not quantitative and not proven.

Chemical process control system engineers agree one needs a valid dynamic process model before designing a feedback controller to hold a desired dependent response variable like temperature about a desirable setpoint target, by adjusting an independent manipulated variable like combustion rate, which strongly affects effects the controlled variable, temperature. Such a control system is called a thermostat. They are in oil refineries, buildings, homes, cars and the human body.

Before designing any control system, engineers analyze four criteria necessary to insure the control system will work at all, even if it is built. There is a rigorous mathematical method for determining the desired setpoint setting, involving optimization of a risky tradeoff¹⁻⁵. This technology is available, but humanity is not organized properly to use it. (We can't agree whether it is too hot or too cold.) Further the system must be measurable, observable and controllable. Mathematical criteria^{6,7} were developed in 1970's to verify a priori whether these conditions are satisfied.

Work⁸⁻¹⁰ in early 1997 indicated the proposed Kyoto thermostat did not satisfy any of these criteria, hence it will never work. (Never. No matter what the global consensus and government research spending may be.) After 20 years the conclusion has been confirmed by the UN and

world scientific community. They recently estimated it will take at least 50 years and \$4 trillion to build their thermostat. Still won't work.

2. MODELING APPROACHES.

There are three main approaches to mathematically describing the atmosphere's behavior. Two encompass forms of the main laws of physics: conservation of energy (First Law of Thermodynamics), conservation of matter, and Stefan-Boltzmann Law of radiators. These laws incorporate the physical properties of the system and rate law parameters. When an input variable change like CO₂ is specified, like a step, ramp, sine wave or arbitrary function, the model can be solved for the output response variables, temperatures.

Rigorous space and time is described by complex coupled nonlinear partial differential equations^{11, 12}. Since the atmosphere varies greatly day to night, from the surface to space, equator to poles, clouds, auroras, storms and lightening, with gases, liquids and solids of different compositions, temperatures and pressures, solving the equations is not practical, even with computers. Weathermen cannot forecast accurately more than a few days in advance. But the forms of these equations falsify the greenhouse gas theory¹².

Control system engineers developed lumped parameter model methods to simplify the mathematics by combining spatial effects with effective properties and parameters, retaining the dynamic behavior. The partial differential equations are reduced to ordinary differential equations which are easier to solve and reveal important system characteristics, like stability, measurability, observability and controllability. A body of control theory for differential equation models was developed since 1960¹³. Multivariable dynamic control systems now model and control complex commercial oil refinery reactors and distillation columns around the world by several businesses. So control system engineering works, adds value.

If only long term steady-state behavior rather than short term dynamic responses is of primary interest, the dynamic terms can be neglected and the system of differential equations reduces to algebraic equations. Over long periods, say years or decades, this steady-state model quantifies output changes caused by input and property changes. This is the scientific cause and effect model missing in climate change studies.

Review of the GHGT literature indicates climate scientists have bypassed the two rigorous engineering methods and opted for an empirical statistical regression input - output modeling approach from atmospheric measurements. These methods can detect possible correlations but never prove causation alone. (Control engineers incorporate this law of science in their work.) Since scientists cannot specify the type of Earth's input functions known to better identify the system with experimental testing, they are limited to measured natural input fluctuations which are weak system identifiers. While this approach may give useful interpolations, it cannot give reliable extrapolations beyond the range of its basis data. This knowhow is often used by engineers, the art of combining science and empiricism to build things. Sadly, UN and university modelers have wasted billions of dollars on this approach,

known to be futile, proving it doesn't work for themselves¹⁴. Yet some continue attempting the impossible.

The second method is employed here to draw conclusions. The key is to model Earth's radiating atmosphere with Martin Hertzberg's general radiant energy transfer rate law¹⁵.

We will derive algebraic expressions that quantify the effect of adding CO₂ on temperature. If we assign physical properties correctly to each radiator we should have accurate radiant energy flows, temperatures and the effect of inputs like CO₂ on all these dependent variables.

3. MATTER AND ENERGY CONSERVATION

The Law of Conservation of matter, energy, inventory, money, human body and everything at every instant, including the atmosphere's energy, is

$$\text{output rate} = \text{input rate} - \text{accumulation rate.} \quad (1)$$

For matter, units are any mass or molar flow rate. For moles of CO₂ in the atmosphere:

Output to plant photosynthesis + ocean absorption/desorption = plant decomposition + forest fires + volcanoes + fossil fuel combustion - change of CO₂ in the atmosphere.

The first term is the photosynthesis biological chemistry reaction neglected by GHGT,

CO₂ + H₂O + sunshine = carbs + sugars + starches + O₂

The rate of reaction consuming CO₂ and solar energy is

$$Q_{pt} = K [\text{CO}_2] [\text{H}_2\text{O}] \exp(-k/T_s)$$

This shows as the concentration [CO₂] increases, sunshine and CO₂ consumption increases, reducing T_s, a cooling effect, and [CO₂], a stabilizing effect. Further as T_s increases, the rate of CO₂ and energy consumption increases, reducing [CO₂] and T_s, a stabilizing, cooling effect. CO₂ is harmless green plant food, essential to flora and fauna.

The CO₂ reaction has an energy rate term Q_{pt} from atmosphere to surface, coupling surface and atmosphere energy balances. It varies with T and [CO₂]. I will simplify, assume energy transfer between them is constant Q_{pt}.

For energy, the units in this paper are w/m² of Earth's spherical surface. w is watts, joules of energy per second. To convert to kw, multiply by the area of Earth's surface, 510*10⁶ km² * kw * 10⁶ m² / 1000 w * km² = 5.1*10¹¹ kw / w/m².

Accumulation rate is A = mCp dT_a/dt / area, where T_a is an effective temperature of the whole atmosphere. This is the First Law of Thermodynamics, conservation of energy. This is an

ordinary differential equation for a lumped parameter system with effective properties. It can be integrated numerically for the transient response of T_a to a change in input rate. It needs the output rate law depending on T_a and T_s .

Steady-state equilibrium occurs by definition when accumulation = $A = 0 = dT_a/dt$.

output rate = input rate.

This simplification converts differential equations to algebraic ones, avoiding integration. This is a good approximation for long term averages with fixed average inputs.

For the atmosphere, energy in = energy absorbed from sun + energy absorbed from surface + heat transfer by conduction and convection from surface + internal generation = $Q_{da} + Q_{sau} + Q_{scu} + Q_{ai}$.

Energy out is energy emitted to space + energy consumed by photosynthesis = $Q_{au} + Q_{pt}$

Atmospheric energy balance is out = in - accumulation

$$Q_{au} + Q_{pt} = Q_{da} + Q_{sau} + Q_{scu} + Q_{ai} - A_a$$

Surface energy balance is

$$Q_{su} + Q_{scu} + Q_{sau} + Q_{sa} = Q_{ds} + Q_{si} + Q_{pt} - A_s$$

where¹⁶

Q_{da} is energy flow down from the sun absorbed by the atmosphere, 78

Q_{sau} is energy flow up from the surface absorbed by the atmosphere, 24

Q_{scu} is energy flow up from the surface to the atmosphere by convection and conduction, from volcanoes, through crust and fossil fuel combustion, 97

Q_{ai} is energy generated within the atmosphere by ionization, electric charge, phase changes

A_a is energy accumulation in the atmosphere

Q_{au} is energy flow emitted by atmosphere to space, 199

Q_{ds} is energy flow down from the sun absorbed by the surface, 161

Q_{su} is energy flow emitted by the surface transmitted by atmosphere directly to space, 40

Q_{si} is energy generated within surface, small

Q_{sa} is net energy absorbed by surface, small

Q_{pt} is solar energy consumed by plant photosynthesis, need to quantify

A_s is energy accumulation in surface.

Atmosphere: $199 = 78 + 24 + 97$

Surface: $24 + 97 + 40 = 161$

Absorbed $78 + 161 =$ emitted to space 239

4. RADIANT ENERGY FLOWS

Rate law expressions for each output Q in terms of physical properties and inputs give the outputs, T_a and T_s , of interest

Radiation impinging on matter is either reflected, transmitted or absorbed. These fractions are reflectivity ρ , transmissivity τ , and absorptivity α . $\rho + \tau + \alpha = 1$ by their definition. They are physical properties of every radiator. Earth's atmosphere with clouds has all three fractions > 0 . Absorptivity has a spectrum of intensity vs. wavelength unique to each atom and molecule. So absorptivity depends on the impinging spectrum and is different for incoming solar and upwelling surface radiation. We will designate them α_{ad} and α_{au} . $\alpha_{ad} + \rho_{ad} + \tau_{ad} = 78/341 + 79/341 + 184/341 = 0.2287 + 0.2317 + 0.5396 = 1$. $\alpha_a = 24 / (24 + 40) = 0.375$.

The solar constant, S , is about 1364 w/m^2 of a disk perpendicular to sun beam at Earth. Since the surface area of a sphere is 4 times the area of its inscribed circle, distributing this radiation for an average over Earth's surface gives average incoming intensity, day and night, of $1364/4 = 341 \text{ w/m}^2$ of Earth's surface.

$Q_{da} = \alpha_{ad} S/4 = 0.2287 * 341 = 78.0 \text{ w/m}^2$ (α_{ad} increases slightly with CO2, CO2 absorption wavelength spectrum left tail overlaps solar spectrum right tail.)

$Q_{scu} =$ about 17 from thermals and 80 from latent heat = 97 w/m^2 . (Q_{scu} does not depend much on CO2.)

Surface reflectivity is $\rho_{sd} = 23/184 = 0.1250$

$Q_{ds} = \tau_{ad} * (1 - \rho_{sd}) S/4 = 0.5396 * (1 - 0.1250) * S/4 = 0.4721 * 1364/4 = 161$

5. RADIATOR LAW

Next we employ the basic law for intensity for all EMR radiators, the Stefan - Boltzmann Law

$$I, \text{ w/m}^2 = 5.67 \epsilon T^4 \quad (2)$$

where I = intensity, power or irradiance, w/m^2 of radiator surface across all wavelengths, $0 < \epsilon < 1$ is the radiator emissivity and $T = K/100$ where K is the radiator temperature, deg Kelvin. The value 5.67 is a fundamental constant of nature, like universal gravity constant, speed of light, mass of neutron, charge of electron, density of water. Like absorptivity, emissivity has a spectrum of intensity vs. wavelength unique to each atom and molecule.

This S-B Law says any radiator with emissivity ϵ and temperature T will radiate with intensity I . This is not the rate of radiant energy transfer from the radiator to its surroundings; that depends on the intensity of the surroundings radiation and the absorptivity of the surroundings and the radiator from the surroundings radiation. Many writers confuse this because it's units are w , a flow rate. But is it an EMR intensity, the driving force for radiant energy transfer.

When developing the incandescent light, Thomas Edison sought a filament with sufficiently high emissivity ϵ that it could radiate 10 w at sufficiently low temperature that it would not burn up. Filaments oxidize as they age and emissivity decreases, getting hotter until they blow out. Bulb vacuum is never perfect and aging is inevitable.

The general law^{15, 17} of radiant energy transfer between radiators 1 and 0 is

$$Q_{10} = 5.67 [\alpha_0 \epsilon_1 T_1^4 - \alpha_1 \epsilon_0 T_0^4] \quad (3)$$

Chemical engineers use the basic law of radiant energy transfer between two radiators from the S-B Law^{15, 17, 18}. Since all matter radiates, this law applies to all matter: light bulbs, flames, animals, gases, stars and galaxies. Hertzberg gives an elegant derivation¹⁵.

When $Q_{10} > 0$, energy flows from 1 to 0; when $Q_{10} < 0$, energy flows from 0 to 1; when $Q_{10} = 0$ there is no energy flow. Both radiators are radiating with identical intensity at each other's surface. The following derivation determines the direction of transfer as an output result from T_s and T_o ; direction is not assumed.

The driving force for radiant energy flow is the intensity difference at a radiator's surface, a vector with direction. This is analogous to pressure drop for fluid flow, voltage drop for electric current, temperature difference for heat transfer by conduction and convection, composition difference for mass transfer and force for mass acceleration, $F = mA$. Flow of energy and matter requires a driving force, a potential, a gradient with a direction, hence a force vector.

When only radiation is involved in a radiator, no electric, mechanical, chemical, thermal, nuclear or other energy transfer mechanisms are involved, then Kirchhoff's Law says the emissivity and absorptivity are equal. Some researchers assume this Law because the rate law is simplified (α and ϵ factor out). Earth's atmosphere does not follow this law. Some researchers make their mathematics even simpler and less applicable by assuming both radiators are black bodies, emissivity = absorptivity = 1.0 (α and ϵ disappear). Beware of any conclusions from those assumptions.

To simplify, some versions of GHGT assume atmosphere and surface obey Kirchhoff's Law:

$$\alpha_0 = \epsilon_0 \text{ and } \epsilon_1 = \alpha_1, \alpha_0 \epsilon_1 = \alpha_1 \epsilon_0$$

Substituting

$$Q_{10} = 5.67 [\alpha_0 \epsilon_1 T_1^4 - \alpha_1 \epsilon_0 T_0^4] = 5.67 [\epsilon_0 \epsilon_1 T_1^4 - \epsilon_1 \epsilon_0 T_0^4] = 5.67 \epsilon_0 \epsilon_1 [T_1^4 - T_0^4]$$

Simplifying further, assume both are black body radiators

$$\alpha_0 = \epsilon_1 = \alpha_1 = \epsilon_0 = 1.0$$

We get a common expression

$$Q_{10} = 5.67 [T_1^4 - T_0^4], > 0 \text{ because } T_1 \text{ was selected } > T_0$$

which is then used to claim radiant energy only transfers from the warmer radiator, T_1 , to the cooler one, T_0 . This may commonly occur, but is not universal. This simplification is only true in general when

$$Q_{10} = 5.67 [\alpha_0 \epsilon_1 T_1^4 - \alpha_1 \epsilon_0 T_0^4] > 0, \text{ or } \alpha_0 \epsilon_1 T_1^4 > \alpha_1 \epsilon_0 T_0^4$$

At equilibrium $Q_{10} = 0$ and $\alpha_0 \epsilon_1 T_1^4 = \alpha_1 \epsilon_0 T_0^4$.

This explains why dissimilar walls in a room at equilibrium may have different temperatures.

6. GLOBAL EARTH RADIATOR

Earth radiates to space on average

$$Q_{\text{out}} = Q_{\text{au}} + Q_{\text{su}} = 5.67 [\alpha_0 \epsilon_1 T_1^4 - \alpha_1 \epsilon_0 T_0^4] \quad (4)$$

Q_{out} is measured to be about 239: 169 from atmosphere gases, 30 from clouds and 40 from the surface. We will simplify by including clouds with atmosphere gases, so $Q_{\text{au}} = 199$ and $Q_{\text{su}} = 40$.

T_1 is the average radiating temperature of the Earth, surface and atmosphere combined. It is typically assumed 278.5667K = +5.4167C. Atmosphere is assumed 255K = -18.15C; surface is assumed 288K = +14.85C.

T_0 is the radiating temperature of space, mostly microwave background at 3.7 deg K

α_0 is absorptivity of space, close to 1 since it does not reflect

α_1 is absorptivity of Earth, hard to measure but deduced $239/341 = 0.70 = 1 - \text{albedo}$

ϵ_0 is emissivity of space, close to 1

ϵ_1 is emissivity of Earth. Hard to estimate. Some assume 0.997 which is pure water. A better value is 0.70, close to its absorptivity by Kirchhoff's Law. We assume radiant energy transfer dominates other energy transfer mechanisms on Earth.

$$Q_{\text{out}} = 5.67 [1 * 0.70 * 2.7857^4 - 0.70 * 1 * 0.037^4] = 5.67 [42.153672 - 0.00000131] = 5.67 [42.153673] = 239.01$$

This shows Earth's special surroundings do not affect its radiant energy transfer to space because space is so cold it emits with negligible intensity. Earth's radiating intensity = energy transfer rate to space, a special case. Earth's absorptivity is of no consequence here because space intensity is negligible. Solar input is handled separately.

We can accurately use $Q_{out} = 5.67 \epsilon_1 T_1^4 = 239$.

We can infer $\epsilon_1 = Q_{out} / 5.67 * T_1^4 = 239/5.67 * 2.7857^4 = 0.69997 = 0.70$

When emitting gases increase in the atmosphere, like CO2 with ϵ about 0.1 replacing a non-emitting gas like O2, $\epsilon = 0$, Earth's emissivity ϵ_1 increases.

Since Q_{out} does not vary with CO2 because it is set by Earth's incoming energy, it follows from S-B Law that T_1 decreases with ϵ_1 . As CO2 increases, Earth becomes a better radiator, emitting the same intensity to space at a lower temperature.

This proves CO2 causes global cooling, however slight.

For the atmosphere, $\epsilon_{au} = Q_{au} / 5.67 * T_a^4 = 199 / 5.67 * 2.55^4 = 0.83006$, mostly clouds.

For the surface, $Q_{sau} + Q_{su} = I_s = 5.67 \epsilon_{su} T_s^4$

Estimate $\epsilon_{su} = (Q_{sau} + Q_{su}) / 5.67 * T_s^4 = (24 + 40) / 5.67 * 2.88^4 = 0.16407$

$\epsilon_1 = f \epsilon_{au} + (1 - f) \epsilon_{su} = 0.70 = f * 0.83006 + (1 - f) * 0.16407$; $f = 0.804712$

Note intensity weighted average is

$f_i = 199/239 = 0.8326$, so the weighting factor is close to intensity ratio.

Assume doubling CO2 increases atmospheric emissivity 1% to $1.01 * 0.83006 = 0.83836$.

$\epsilon_1 = f \epsilon_{au} + (1 - f) \epsilon_{su} = 0.804712 * 0.83836 + (1 - 0.804712) * 0.16407 = 0.70668$

$T_1^4 = Q_{out} / 5.67 \epsilon_1 = 239 / 5.67 * 0.70668 = 59.64753$, $T_1 = 277.9061$ or $4.7561C$, down from $5.4167C$ by $-0.6606C$

7. SURFACE RADIATOR

Surface radiates with intensity

$I_s = 5.67 \epsilon_{su} T_s^4 = 5.67 * 0.16407 * 2.88^4 = 64$ at $14.85C$

Energy transfer from surface is transmitted through the atmosphere directly to space, reflected back down from clouds and dust, and absorbed by atmospheric gas and clouds.

$Q_s = Q_{su} + Q_{sr} + Q_{sau}$

$Q_s = \tau_{au} I_s + \rho_{au} I_s + Q_{sau} = (\tau_{au} + \rho_{au}) I_s + Q_{sau} = (1 - \alpha_{au}) I_s + Q_{sau} = 5.67 (1 - \alpha_{au}) \epsilon_{su} T_s^4 + Q_{sau}$

$$Q_s = 5.67 (1 - 24/64) 0.16407 * 2.88^4 + Q_{sau} = 40 + Q_{sau}$$

I assume $\rho_a = 0$, neglect atmospheric reflection down as most GHGT authors do, which simplifies the model but may introduce small error.

8. SURFACE RADIATION ABSORBED BY ATMOSPHERE

The rate of radiant energy transferred from the surface to and absorbed by the atmosphere is

$$Q_{sau} = 5.67 [\alpha_{aux} \epsilon_{su} T_s^4 - \alpha_{su} \epsilon_{ad} T_a^4] \quad (6)$$

Assume 3.0 of 97 thermals and latent heat is absorbed as radiation by atmosphere.

Estimate $\alpha_{aux} = (24 + 3.0) / (64 + 3.0) = 0.40298$,

Assume $\alpha_{su} = 0.1$ $\alpha_{sd} = 0.1 * 161/184 = 0.1 * 0.875 = 0.0875$

Assume $\epsilon_{ad} = 0.1$ $\epsilon_{au} = 0.1 * 0.83006 = 0.083006$

$$Q_{sau} = 5.67 [0.40298 * 0.16407 * 2.88^4 - 0.0875 * 0.083006 * 2.55^4]$$

$$Q_{sau} = 5.67 [4.5487 - 0.3071] = 5.67 * 4.2416 = 24.0500. \text{ Checks with } 24.$$

An important observation here. GHGT promoters say increased CO2 heats the surface. Looking at this rate law in isolation one sees CO2 increases atmospheric emissivity ϵ_{ad} which is a resistance to energy transfer. If all other terms are constant, surface radiator temperature T_s must compensate and rise, causing global warming. Their error is their assumption the other terms are constant is not correct. Atmospheric temperature T_a is not a constant, it decreases with CO2 as proved above. That reduces the resistance to transfer, a cooling effect compensating more or less for the emissivity increase warming effect. Further, radiant heat transferred from the surface Q_{sau} is also not constant, it drops as CO2 absorbs some incoming solar energy by its absorption bands 2.2, 2.8, 4.2 micrometers, however slight, another cooling effect. So from the rate law alone we find one warming and two cooling effects. The net could go either way.

Substituting into the surface energy out expression

$$Q_s = 5.67 (1 - \alpha_{au}) \epsilon_{su} T_s^4 + 5.67 [\alpha_{aux} \epsilon_{su} T_s^4 - \alpha_{su} \epsilon_{ad} T_a^4] = 5.67 [(1 - \alpha_{au}) \epsilon_{su} + \alpha_{aux} \epsilon_{su}] T_s^4 - 5.67 \alpha_{su} \epsilon_{ad} T_a^4$$

$$Q_s = 5.67 [\epsilon_{su} (1 - \alpha_{au} + \alpha_{aux}) T_s^4 - \alpha_{su} \epsilon_{ad} T_a^4] = 5.67 [\epsilon_{su} (1 - \alpha_{au} + \alpha_{aux}) T_s^4 - \alpha_{su} \epsilon_{ad} T_a^4]$$

$$Q_s = 5.67 [\epsilon_{su} (1 - \alpha_{au} + \alpha_{aux}) T_s^4 - \alpha_{su} \epsilon_{ad} T_a^4] \quad (7)$$

$$Q_s = 5.67 [0.16407 (1 - 0.375 + 0.40298) 2.88^4 - 0.0875 * 0.083006 * 2.55^4]$$

$$Q_s = 5.67 [0.16407 (70.722) - 0.3071] = 64.05. \text{ Checks with } 64.$$

The surface energy balance is

$$Q_{su} + Q_{scu} + Q_{opt} + Q_{sau} + Q_{sa} = Q_{ds} + Q_{si} - A_s$$

$$Q_s = Q_{su} + Q_{sau} = Q_{ds} - Q_{scu} - Q_{pt} + Q_{si} - Q_{sa} - A_s$$

Substituting into surface energy balance eqn (1)

$$5.67 [\epsilon_{su} (1 - \alpha_{au} + \alpha_{aux}) T_s^4 - \alpha_{su} \epsilon_{ad} T_a^4] = \tau_{ad} * (1 - \rho_{sd}) S/4 - Q_{scu} - Q_{pt} + Q_{si} - Q_{sa} - A_s \quad (8)$$

$$5.67 [0.16407 (1 - 0.375 + 0.40298) 2.88^4 - 0.0875 * 0.083006 * 2.55^4] = 0.5396 * (1 - 0.1250) * 1364/4 - 97 - Q_{pt} + Q_{si} - Q_{sa} - A_s$$

$$5.67 [0.16407 * 1.02798 * 2.88^4 - 0.007263 * 2.55^4] = 0.1180 * 1364 - 97 - Q_{pt} + Q_{si} - Q_{sa} - A_s$$

$$64.050 = 63.952 - Q_{pt} + Q_{si} - Q_{sa} - A_s$$

The left side of eqn (8) depends on two variables, T_s and T_a . The only property parameters that are influenced by CO2 are atmosphere absorptivity and emissivity, α_a and ϵ_a . Right side terms are independent inputs.

9. ATMOSPHERE RADIATOR

The atmospheric energy balance will give another independent relationship among these two variables and system property parameters that change with CO2.

$$Q_{au} = Q_{da} + Q_{sau} + Q_{scu} + Q_{ai} - A_a$$

Rearranging for T on the left,

$$Q_{au} - Q_{sau} = Q_{da} + Q_{scu} + Q_{ai} - A_a$$

$$Q_{au} = 5.67 \epsilon_{au} T_a^4 = 5.67 * 0.83006 * 2.55^4 = 199$$

$$Q_{da} = \alpha_{ad} S/4 = 0.2287 * 1364/4 = 78.0$$

$$Q_{sau} = 5.67 [\alpha_{aux} \epsilon_{su} T_s^4 - \alpha_{su} \epsilon_{ad} T_a^4] = 5.67 [0.40298 * 0.16407 * 2.88^4 - 0.0875 * 0.083006 * 2.55^4] = 5.67 [4.5487 - 0.3071] = 5.67 * 4.2416 = 24.0500. \text{ Checks with } 24.$$

Substituting into atmosphere energy balance eqn (1)

$$199 - 24 = 78 + 97 + Q_{pt} + Q_{ai} - A_a$$

$$175 = 175 + Q_{pt} + Q_{ai} - A_a = 175 + 0 + 0 - 0$$

$$5.67 \epsilon_{au} T_a^4 - 5.67 [\alpha_{aux} \epsilon_{su} T_s^4 - \alpha_{su} \epsilon_{ad} T_a^4] = \alpha_{ad} S/4 + Q_{scu} + Q_{pt} + Q_{ai} - A_a$$

$$5.67 [(\epsilon_{au} + \alpha_{su} \epsilon_{ad}) T_a^4 - \alpha_{aux} \epsilon_{su} T_s^4] = \alpha_{ad} S/4 + Q_{scu} + Q_{pt} + Q_{ai} - A_a \quad (9)$$

$$5.67 (0.83006 + 0.0875 * 0.083006) 2.55^4 - 5.67 * 0.40298 * 0.16407 * 2.88^4 = 0.2287 * 1364/4 + 97 + Q_{pt} + Q_{ai} - A_a$$

$$5.67 (0.8373) 2.55^4 - 5.67 (0.06612) 2.88^4 = 78 + 97 + Q_{pt} + Q_{ai} - A_a$$

$$200.736 - 25.79 = 175 + Q_{pt} + Q_{ai} - A_a$$

174.946 = 174.987 + 0 + 0 + 0. Checks.

10. CONNECT SURFACE AND ATMOSPHERE RADIATORS

Now we have two coupled linear differential equations (8) and (9) for two unknowns T_s^4 and T_a^4 . They can be integrated simultaneously for transient responses $T_a(t)$ and $T_s(t)$ to any specified inputs. Setting accumulation derivative terms A_a and $A_s = 0$, we have two linear algebraic equations for steady-state solutions T_a^4 and T_s^4 . They can be solved iteratively on any calculator. Just specify inputs and properties.

Find a single equation for T_s^4 by substituting eqn (9) into (8) and eliminating T_a^4 .

$$5.67 (\epsilon_{au} + \alpha_{su} \epsilon_{ad}) T_a^4 = 5.67 \alpha_{aux} \epsilon_{su} T_s^4 + \alpha_{ad} S/4 + Q_{scu} + Q_{pt} + Q_{ai} - A_a \quad (9)$$

$$5.67 T_a^4 = [5.67 \alpha_{aux} \epsilon_{su} T_s^4 + \alpha_{ad} S/4 + Q_{scu} + Q_{pt} + Q_{ai} - A_a] / (\epsilon_{au} + \alpha_{su} \epsilon_{ad})$$

$$5.67 2.55^4 = [5.67 * 0.40298 * 0.16407 * 2.88^4 + 0.2287 * 1364/4 + 97 + Q_{pt} + Q_{ai} - A_a] / (0.83006 + 0.0875 * 0.083006)$$

$$5.67 2.55^4 = [0.374883 * 2.88^4 + 0.2287 * 1364/4 + 97 + Q_{pt} + Q_{ai} - A_a] / 0.837323$$

$$239.742 = 239.785 + [Q_{pt} + Q_{ai} - A_a] / 0.837323. \text{ Checks.}$$

$$5.67 \epsilon_{su} (1 - \alpha_{au} + \alpha_{aux}) T_s^4 - 5.67 \alpha_{su} \epsilon_{ad} T_a^4 = \tau_{ad} * (1 - \rho_{sd}) S/4 - Q_{scu} + Q_{pt} + Q_{si} - Q_{sa} - A_s \quad (8)$$

$$5.67 \epsilon_{su} (1 - \alpha_{au} + \alpha_{aux}) T_s^4 - \alpha_{su} \epsilon_{ad} [5.67 \alpha_{aux} \epsilon_{su} T_s^4 + \alpha_{ad} S/4 + Q_{scu} + Q_{pt} + Q_{ai} - A_a] / (\epsilon_{au} + \alpha_{su} \epsilon_{ad}) = \tau_{ad} * (1 - \rho_{sd}) S/4 - Q_{scu} - Q_{pt} + Q_{si} - Q_{sa} - A_s \quad (9) \text{ in } (8)$$

$$5.67 \epsilon_{su} [(1 - \alpha_{au} + \alpha_{aux}) - (\alpha_{su} \epsilon_{ad} \alpha_{aux}) / (\epsilon_{au} + \alpha_{su} \epsilon_{ad})] T_s^4 - \alpha_{su} \epsilon_{ad} [\alpha_{ad} S/4 + Q_{scu} + Q_{pt} + Q_{ai} - A_a] / (\epsilon_{au} + \alpha_{su} \epsilon_{ad}) = \tau_{ad} * (1 - \rho_{sd}) S/4 - Q_{scu} - Q_{pt} + Q_{si} - Q_{sa} - A_s$$

$$5.67 \epsilon_{su} [1 - \alpha_{au} + \alpha_{aux} - (\alpha_{su} \epsilon_{ad} \alpha_{aux}) / (\epsilon_{au} + \alpha_{su} \epsilon_{ad})] T_s^4 = - Q_{scu} - Q_{pt} + Q_{si} - Q_{sa} - A_s + \alpha_{su} \epsilon_{ad} [\alpha_{ad} S/4 + Q_{scu} + Q_{pt} + Q_{ai} - A_a] / (\epsilon_{au} + \alpha_{su} \epsilon_{ad}) + \tau_{ad} * (1 - \rho_{sd}) S/4 \quad (10)$$

This is the goal, a single equation for T_s from energy inputs and Earth's radiating properties. This is a linear ordinary differential equation with analytic transient solutions for step and ramp inputs. Set $A_a = A_s = 0$ for the algebraic equation for the steady-state solution for T_s .

Six parameters, ϵ_{su} , ϵ_{ad} , α_{au} , α_{aux} , α_{ad} , τ_{ad} , vary with CO2.

$$\epsilon_{au} = 0.830060$$

$$\epsilon_{ad} = 0.083006$$

$$\epsilon_{su} = 0.16407$$

$$\alpha_{au} = 0.375$$

$$\alpha_{aux} = 0.40298$$

$$\alpha_{su} = 0.0875$$

$$\alpha_{ad} = 0.22874$$

$$\alpha_{sd} = 0.875$$

$$\tau_{ad} = 1 - 78/341 - 79/341 = 1 - 0.22874 - 0.23167 = 0.539589$$

$$\rho_{sd} = 0.1250$$

$$5.67 * 0.16407 [1 - 0.375 + 0.40298 - (0.0875 * 0.083006 * 0.40298) / (0.83006 + 0.0875 * 0.083006)] T_s^4 = 0.0875 * 0.083006 [0.22874 * 1364/4 + Q_{scu} + Q_{pt} + Q_{ai} - A_a] / (0.83006 + 0.0875 * 0.083006) + 0.539589 * (1 - 0.1250) * 1364/4 - Q_{scu} - Q_{pt} + Q_{si} - Q_{sa} - A_s$$

$$5.67 * 0.16407 [1.02798 - (0.002927) / (0.837323)] T_s^4 = 0.007263 [0.22874 * 1364/4 + Q_{scu} + Q_{pt} + Q_{ai} - A_a] / (0.837323) + 0.47214 * 1364/4 - Q_{scu} - Q_{pt} + Q_{si} - Q_{sa} - A_s$$

$$5.67 * 0.16407 [1.02798 - 0.0034955] T_s^4 = 0.0086741 [0.22874 * 1364/4 + Q_{scu} + Q_{pt} + Q_{ai} - A_a] + 0.47214 * 1364/4 - Q_{scu} - Q_{pt} + Q_{si} - Q_{sa} - A_s$$

$$5.67 * 0.16407 [1.024485] T_s^4 = 0.001941 * 1364/4 + 0.47214 * 1364/4 + 0.0086741 (Q_{scu} + Q_{pt} + Q_{ai} - A_a) - Q_{scu} - Q_{pt} + Q_{si} - Q_{sa} - A_s$$

$$0.953054 T_s^4 = 0.474111 * 1364/4 - 0.991326 (Q_{scu} + Q_{pt}) + 0.0086741 (Q_{ai} - A_a) + Q_{si} - Q_{sa} - A_s$$

$$T_s^4 = [0.474111 * 1364/4 - 0.991326 (97 + Q_{pt}) + 0.0086741 (Q_{ai} - A_a) + Q_{si} - Q_{sa} - A_s] / 0.953054$$

$$T_s^4 = [161.67185 - 96.15862 + 0 + 0 - 0 - 0] / 0.953054 = 65.51324 / 0.953054 = 68.74032 = 2.879406^4; T_s = 287.9406K = 14.7906C. Excel is 14.7958C. Checks with 14.85C.$$

Finally, T_a is determined directly from T_s , eqn (9)

$$T_a^4 = [5.67 \alpha_{aux} \epsilon_{su} T_s^4 + \alpha_{ad} S/4 + Q_{scu} + Q_{pt} + Q_{ai} - A_a] / 5.67 (\epsilon_{au} + \alpha_{su} \epsilon_{ad}) \quad (9)$$

$$T_a^4 = [5.67 * 0.40298 * 0.16407 * 2.879406^4 + 0.22874 * 1364/4 + 97 + Q_{pt} + Q_{ai} - A_a] / 5.67 (0.83006 + 0.0875 * 0.083006)$$

$$T_a^4 = [0.37488 * 2.879406^4 + 0.22874 * 1364/4 + 97 + Q_{pt} + Q_{ai} - A_a] / 5.67 (0.87323)$$

$$T_a^4 = [0.37488 * 68.74032 + 77.987 + 97 + Q_{pt} + Q_{ai} - A_a] / 4.7476$$

$$T_a^4 = [25.7694 + 77.987 + 97 + Q_{pt} + Q_{ai} - A_a] / 4.7476$$

$$T_a^4 = [200.756 + 0 + 0 - 0] / 4.7476 = 42.28586. T_a = 255.005K = -18.145C. Excel - 18.1403C$$

Use the alternative equation addition method for easier hand calculation. Find a single equation for T_s^4 by adding eqns (8) and (9), eliminating T_a^4 .

$$A T_s^4 - B T_a^4 = C \quad (8)$$

$$X T_a^4 - Y T_s^4 = Z \quad (9)$$

Multiply eqn (8) by X/B

$$AX/B T_s^4 - X T_a^4 = CX/B \quad (8)$$

Add (8) to (9) to eliminate T_a^4

$$AX/B T_s^4 - Y T_s^4 = Z + CX/B \quad (8) \& (9)$$

$$(AX/B - Y) T_s^4 = Z + CX/B$$

$$T_s^4 = (Z + CX/B) / (AX/B - Y) \quad (10)$$

$$T_a^4 = (Z + Y T_s^4) / X \quad (9)$$

$$A = 5.67 \epsilon_{su} (1 - \alpha_{au} + \alpha_{aux})$$

$$B = 5.67 \alpha_{su} \epsilon_{ad}$$

$$C = \tau_{ad} * (1 - \rho_{sd}) S/4 - Q_{scu} - Q_{pt} + Q_{si} - Q_{sa} - A_s$$

$$X = 5.67 (\epsilon_{au} + \alpha_{su} \epsilon_{ad})$$

$$Y = 5.67 \alpha_{aux} \epsilon_{su}$$

$$Z = \alpha_{ad} S/4 + Q_{scu} + Q_{pt} + Q_{ai} - A_a$$

$$A = 5.67 * 0.16407 (1 - 0.375 + 0.40298) = 5.67 * 0.16407 * 1.02798 = 0.956306$$

$$B = 5.67 * 0.0875 * 0.083006 = 0.041181$$

$$C = 0.539589 (1 - 0.1250) * 1364/4 - 97 - 0 + 0 - 0 - 0 = 161.00 - 97 = 64.00$$

$$X = 5.67 (0.83006 + 0.0875 * 0.083006) = 5.67 (0.83006 + 0.007263) = 5.67 (0.83732) = 4.74762$$

$$Y = 5.67 * 0.40298 * 0.16407 = 5.67 (0.06612) = 0.37488$$

$$Z = 0.22874 * 1364/4 + 97 + 0 + 0 - 0 = 78 + 97 = 175.00$$

$$T_s^4 = [175 + (64 * 4.74762) / (0.041181)] / [0.956306 * 4.74762 / (0.041181) - 0.37488]$$

$$= (175 + 7,378.35) / (110.2493 - 0.37488) = 7,553.35 / 109.8744 = 68.74528$$

$$T_s = 287.9458 = 14.7958C. \text{ Excel } 14.7958C. \text{ Close to } 14.85C.$$

$$T_a^4 = (175 + 0.37488 * 68.74528) / 4.74762 = (175 + 25.7712) / 4.74762 = 42.288816$$

$$T_a = 255.0095 = -18.1405C. \text{ Excel } -18.1404C. \text{ Close to } -18.15C$$

Increasing CO2 increases α_{au} , ϵ_{ad} , ϵ_{au} . By inspection of eqn (10), when α_{au} goes up, A goes down so T_s goes up. When ϵ_{ad} goes up, B goes up in numerator and denominator, T_s could go either way. When ϵ_{au} goes up, X goes up and T_s could go either way. The net effect depends on the effect of CO2 on these three atmospheric properties.

The remaining work is to use known properties of gas mixtures to determine the atmosphere's properties α_a and ϵ_a . Commercial chemical engineering methods are available for this^{17,18}. The key relationship is the change in properties with atmospheric CO2.

This model is valid for every planet, star and sphere.

11. SENSITIVITY EXAMPLE

Set up a base case solution with estimated inputs and parameters and solve for base case T_s , T_a as above. Then change any input or parameter to determine the change in T_s , T_a .

I am unable to determine the effect of doubling CO2 from 400 to 800 ppmv on the radiating properties of the atmosphere. There are several commercial sources¹⁸ of this proprietary technology. As an example, I will assume CO2 doubling causes atmospheric properties to increase + 1%.

$$\epsilon_{au} = 0.830060 * 1.01 = 0.83836$$

$$\epsilon_{ad} = 0.083006 * 1.01 = 0.083836$$

$$\epsilon_{su} = 0.16407$$

$$\alpha_{au} = 0.375 * 1.01 = 0.37875$$

$$\alpha_{aux} = 0.40298 * 1.01 = 0.40701$$

$$\alpha_{su} = 0.0875$$

$$\alpha_{ad} = 0.22874 * 1.01 = 0.231026$$

$$\alpha_{sd} = 161/184 = 0.875$$

$$\tau_{ad} = 1 - 78/341 * 1.01 - 79/341 = 1 - 0.23103 - 0.23167 = 0.53730 \text{ down from } 0.539589$$

$$\rho_{sd} = 0.1250$$

$$T_s^4 = (Z + CX/B) / (AX/B - Y) \quad (10)$$

$$A = 5.67 \epsilon_{su} (1 - \alpha_{au} + \alpha_{aux})$$

$$B = 5.67 \alpha_{su} \epsilon_{ad}$$

$$C = \tau_{ad} * (1 - \rho_{sd}) S/4 - Q_{scu} - Q_{pt} + Q_{si} - Q_{sa} - A_s$$

$$X = 5.67 (\epsilon_{au} + \alpha_{su} \epsilon_{ad})$$

$$Y = - 5.67 \alpha_{aux} \epsilon_{su}$$

$$Z = \alpha_{ad} S/4 + Q_{scu} + Q_{pt} + Q_{ai} - A_a$$

$$A = 5.67 * 0.16407 (1 - 0.37875 + 0.40701) = 5.67 * 0.16407 * 1.01951 = 0.956566$$

$$B = 5.67 * 0.0875 * 0.083836 = 0.041593$$

$$C = 0.53730 (1 - 0.1250) * 1364/4 - 97 - 0 + 0 - 0 - 0 = 160.3175 - 97 = 63.3175$$

$$X = 5.67 (0.83836 + 0.0875 * 0.083836) = 5.67 (0.83836 + 0.007336) = 5.67 (0.845696) = 4.7951$$

$$Y = 5.67 * 0.40701 * 0.16407 = 5.67 (0.066778) = 0.378635$$

$$Z = 0.231026 * 1364/4 + 97 + 0 + 0 - 0 = 78.78 + 97 = 175.78$$

$$T_s^4 = [(175.78 + (63.3175 * 4.7951) / (0.041593))] / [0.956566 * 4.7951 / 0.041593 - 0.378635]$$

$$= (175.78 + 7,299.61) / (110.2789 - 0.37863) = 7,475.39 / 109.90 = 68.0198$$

$$T_s = 287.1830K = 14.0331C. \text{ Excel } 14.0333C \text{ So change from } 14.7958C \text{ is } -0.76274C.$$

So when atmospheric radiation parameters increase by 1%, T_s decreases from 14.7958C to 14.0331C = - 0.76274C.

$$T_a^4 = (Z + Y T_s^4) / X \quad (9)$$

$$T_a^4 = (175.78 + 0.378635 * 68.0198) / 4.7951 = (175.78 + 25.75468) / 4.7951 = 42.0293$$

$$T_a = 254.6174K = -18.53263C. \text{ Excel } -18.53256. \text{ Close to } -18.15C$$

So globe = 4.7561C - 5.4167C = - 0.6606C

Surface = 14.0333C - 14.7958C = - 0.7625C

Atmosphere = - 18.5326C - (-18.1404C) = -0.3922C

The reason these are not consistent is +1% on all radiator parameters is not the same as +1% on atmospheric emissivity. Absorptivity mitigates emissivity. But all changes are < 0. The surface temperature does not change as much as the atmosphere.

This confirms one of several reasons a human-made thermostat for the Earth adjusting fossil fuel combustion cannot work, the process sensitivity is much too low. The Earth already has a natural thermostat which is very strong and stable. There are no tipping points or runaway instabilities.

12. PERSPECTIVE

With this chemical engineering perspective, I close with a brief geological perspective. Ice ages last about 90k years, separated by 20k year-long warming periods. These 110k year glacial periods caused by solar cycles have occurred regularly for several billion years. Humans have walked the Earth for at least 3 million years or $3,000,000/110,000 = 27$ cycles. It is only during the current 20k year warming period that humanity became civilized, beginning about 14k years ago. This warming period began about 17k years ago. Russian geologists are now predicting the next ice age is upon us, geologically speaking. Warm period has only 3k years left. The ice sheet can be about 2km thick over NYC. UN promotes population control; 7.4 billion humans heading for 11.2 in 2100 is quite enough. If the UN just disbands, their dreams will be fulfilled naturally, in spades.

13. CONCLUSION

The effect of changing atmospheric properties like emissivity due to changing CO₂ on temperature of Earth's surface and atmosphere is derived from two laws of physics: conservation of energy and radiator intensity with temperature. Analytic algebraic equations are derived for each temperature. Specifying radiator physical properties of Earth's surface and atmosphere provides a rigorous algebraic model for directly determining the change of both temperatures from changes in system properties and energy inputs.

Known energy rates¹⁶ and property estimates are used to estimate both temperatures, which check with accepted values.

Assuming a 1% increase in atmosphere properties from doubling CO₂ gives surface temperature drop from 14.786C to 14.033C by - 0.763C. Atmosphere temperature drops from - 18.141C to - 18.533C by - 0.392C.

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