Back-radiation of heat does not exist

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Introduction

In **[1, 2]** the author has analyzed the heat evacuation from the planet according to the one-way heat flow formulation, without the back-radiation of heat according to the two-way heat flow hypothesis of Prevost from back in 1791.

Claes Johnson **[3]** is probably the first scientist who has drawn attention to the unphysical behavior of back-radiation of heat, heralded by IPCC (International Panel for Climate Change) obtained by a two-way heat flow formulation .

Matthias Kleespies **[4]** has given an historical overview on the issue.

In Appendix **1** and in **[1]** the author, by means of simple examples on stacks of semi-transparent and fully opaque infinitely thin slabs, has shown that the two-way heat flow formulation, although giving the same temperature distribution as the one-way heat flow formulation, gives rise to spurious absorption in the slabs. For two slabs a two fold spurious absorption (see Appendix **1**) and for a stack of **N** slabs the spurious absorption in slab **i** is **2(N-i)** as is shown in **[1].**

Back-radiation of heat defenders claim that it would be quite possible that in such infinitely thin slabs, which are a kind of Dirac-type bodies with no volume but two faces, that such absorptions occur.

In this paper are given examples of radiation between two slabs of finite thickness taking into account conduction in the slabs.

The claimed spurious absorption in the infinitely thin slabs can not be hidden anymore for slabs with a finite thickness.

Simple geometry with slabs of finite thickness.

We consider two slabs of **finite** thickness, separated by a vacuum space.

Figure 1

q → | slab1 | vacuum | slab2 | → σ T(4)^4 to outer space

geometry	x(1)	x(2)	x(3)	x(4)
temperatures	T(1)	T(2)	T(3)	T(4)

The geometry is described by 4 parameters: x(1), x(2), x(3), x(4)

The nodal temperatures are indicated by: T(1), T(2), T(3), T(4)

An analysis is carried out by giving a heat input in slab 1 on face 1: q.

An alternative analysis is carried out with q=0 but by giving node x(2) a prescribed temperature T2.
In both cases, the face x(4) of slab 2 radiates to outer space: σT(4)^4

In slab 1 the heat transport is governed by conduction given by the Fourier law:

$$q = - k\,dT/dx \qquad W/m^2 \qquad \textbf{(1)}$$

with k conductivity coefficient W/m/K

In slab 2 a similar equation for conduction is valid**.**
In the slabs the temperature is linear for a conductivity coefficient k constant.

The heat transport by radiation in the vacuum between the slabs is governed by the Stefan-Boltzmann equation. For T(2)>T(3):

$$q2\rightarrow3 = \sigma\,(T(2)^4 - T(3)^4) \qquad q3\rightarrow2 = 0 \qquad \textbf{(2)}$$

with Stefan-Boltzmann constant σ = 5.67 e-8 W/m^2/K^4

The thickness of the vacuum space can be arbitrary, the heat transport by radiation is independent of the distance.

We will take in the numerical examples x(3)-x(2) equal to 1 meter for plotting reasons only, and the slabs have a thickness to be defined by input.
Equation **(2)** can represent both the one way heat flow and the two-way heat flow by radiation. In the one-way heat flow description the heat flux from warm to cold is q2→3 and the heat flow from cold to warm q3→2 = 0.

In the two-way heat flow formulation slab 1 from face 2 emits a quantity σT(2)^4**,** as if the face were looking to zero K in outer-space, according to the Prevost hypothesis.

However, before arriving in outer-space, face 3 of slab2 absorbs heat corresponding to the emission $\sigma T(2)^4$ by face 2 of slab 1.
In the two-way heat flow formulation, according to the Prevost hypothesis, slab 2 emits a quantity $\sigma T(3)^4$, as if the surface were looking to outer-space. Equally, before arriving in outer-space, face 2 of slab 3 absorbs heat corresponding to the emission $\sigma T(3)^4$ by face 3 of slab 2.
According to Prevost, the difference of the hypothetical fluxes is the net heat flux $q2 \rightarrow 3$ as defined by (2).

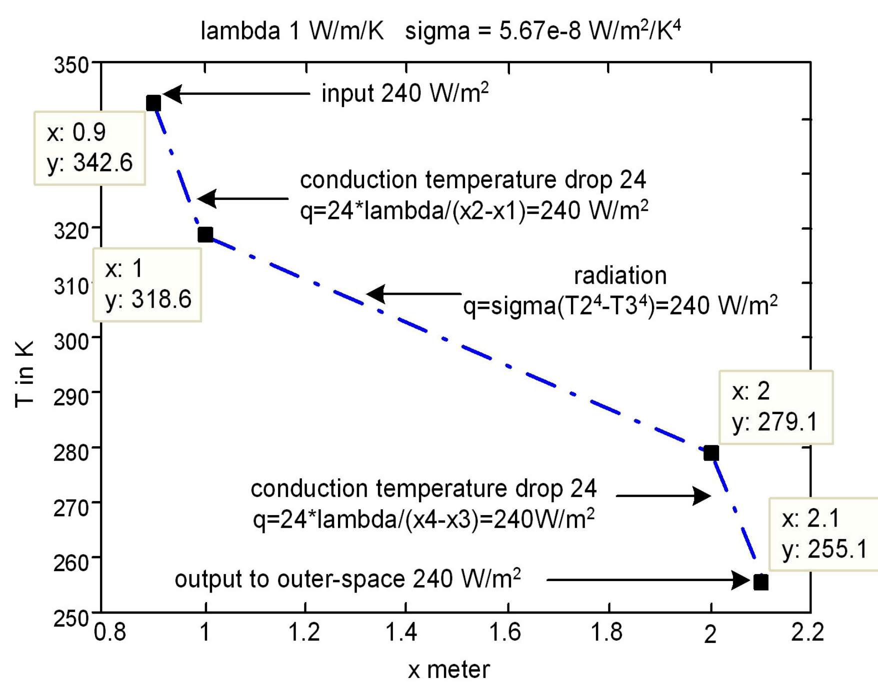
Numerical results

A finite element model (appendix 3) is implemented in MATLAB to solve the equations and to plot the results.
In the numerical runs the thickness of both slabs is 0.1 m.
The conductivity k is taken as 1 W/m/K, Stefan-Boltzmann constant is σ = 5.67e-8

Example with heat input

Run with heat input q = 240 W/m^2 at x=0.9.
The program calculates the 4 temperatures.
The results are given in figure 2.

**Figure 2. Heat input at x(1) and radiation to outer-space at x(4).
Temperatures and heat fluxes are continuous at interfaces.**



The temperature profile is dotted: in the slab the temperature is real resulting from molecular collisions, in vacuum there is no temperature only electromagnetic waves.
We see no back-radiation of heat.

What is more important, there is no place for it in the figure with a continuous temperature profile and a continuous heat flux at the interfaces at x=1 and x=2.

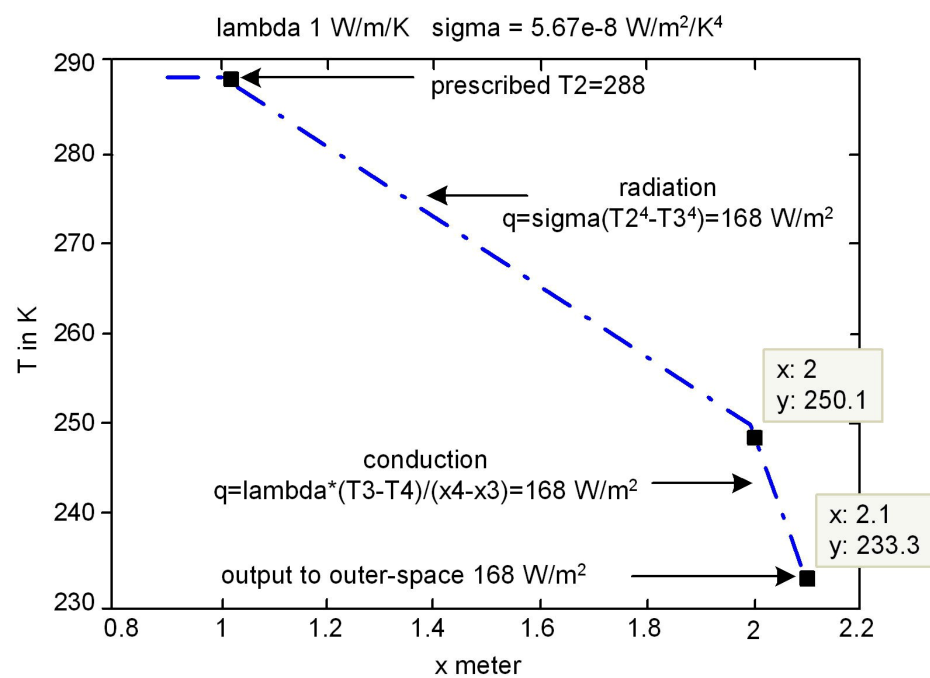
Clearly, there is a one-way heat flux of 240 W/m^2, in the conduction regions and in the radiation region.

The faces of the two slabs exchange electromagnetic information concerning there temperatures and as a consequence a heat flux is established, depending on the two temperatures according to Stefan-Boltzmann (**2**), from the warmer temperature towards the lower one. No heat is flowing from the cold face towards the warm face.
In the left slab the slope of the temperature indicates that there is a flux of 240 W/m^2 due to conduction, giving rise to the same radiation flux of 240 W/m^2, which is absorbed in the right slab where the conduction slope is also 240W/m^2. We see that slab 2 absorbs q=240 and emits in steady-state conditions q=240.
For the two-way formulation a spurious absorption 2q results, as is shown in Appendix **1.**

Example with prescribed temperature.

A prescribed temperature T2 =288 K in x(2) is applied instead of a heat input at x(1).
The prescribed temperature T2 causes automatically a heat input at x(2) .
The program calculates the resulting heat flux as well as the three other temperatures.
The results are given in figure 3.

Figure 3. Prescribed temperature T(2) and radiation to outer-space at x(4).
Temperatures and heat fluxes are continuous at interfaces



The temperature profile is dotted. In the slabs the temperature is real and is caused by molecular collision, in vacuum there is no temperature only electromagnetic waves.

In figure 3 the temperature T2 is prescribed : 288 K.

The heat flux into slab 1 at the point x(2) is calculated by the program: 168 W/m^2.
It leaves the slab immediately at x(2) according to the one-way heat flow formulation by Stefan-Boltzmann **(2).**

No temperature slope is seen in the left slab, since the face at x(1) is isolated, slab 1 is isothermal.

The gradient in slab 2 corresponds also to 168 W/m^2 as indicated by the Fourier law.
Again we note that there is no back-radiation of heat.

But what is important is that the figure shows clearly that back-radiation of heat cannot exist.

There is simply no place for it in the continuous temperature profile and the continuous heat flux at the interfaces.

Another engineering proof that back-radiation of heat does not exist

In the engineering practice it occurs that radiation of heat has to be analyzed between surfaces with different emission coefficients ε.
For example, in case face 2 at x(2) in figure 1 has a emission coefficient ε_2 which is different from the face 3 coefficient ε_3 at x(3), then we introduce corrected Stefan-Boltzmann constants:

$$\sigma_2 = \varepsilon_2\sigma \qquad \text{and} \qquad \sigma_3 = \varepsilon_3\sigma$$

The corrected Stefan-Boltzmann law **(2)** for T(2) >T(3) becomes:

$$q2\rightarrow3 = \sigma_{23}\,(T(2)^4 - T(3)^4) \qquad q3\rightarrow2 = 0 \qquad \textbf{(2a)}$$

For T(3) >T(2)

$$q3\rightarrow2 = \sigma_{32}\,(T(3)^4 - T(2)^4) \qquad q2\rightarrow3 = 0 \qquad \textbf{(2b)}$$

The effective Stefan-Boltzmann constants are calculated from:

$$\sigma_{23} = \sigma_{32} = \varepsilon_{23}\sigma = \varepsilon_{32}\sigma \qquad 1/\varepsilon_{23} = 1/\varepsilon_{32} = 1/\varepsilon_2 + 1/\varepsilon_3 - 1 \qquad \textbf{(3)}$$

(see Appendix **2**)

These relations show clearly what is the practice in the engineering community: one-way heat flow by radiation, from warm to cold and no back-radiation of heat from cold to warm. The amount of heat which radiates from warm to cold depends on both temperatures and on both surface conditions. Electromagnetic information about temperature and surface condition is exchanged between face 2 and face 3. But no heat is flowing from cold to warm!

In the figures 4 and 5 the examples of figure 2 and 3 are repeated, but for different surface conditions.

Figure 4

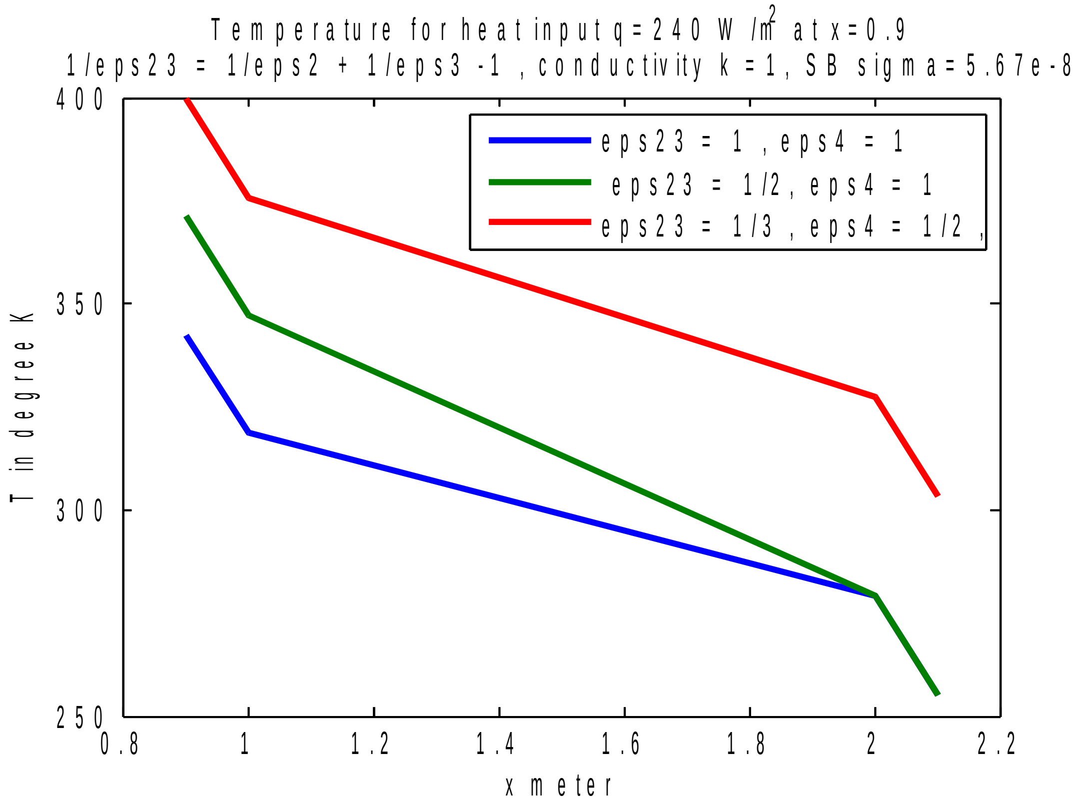
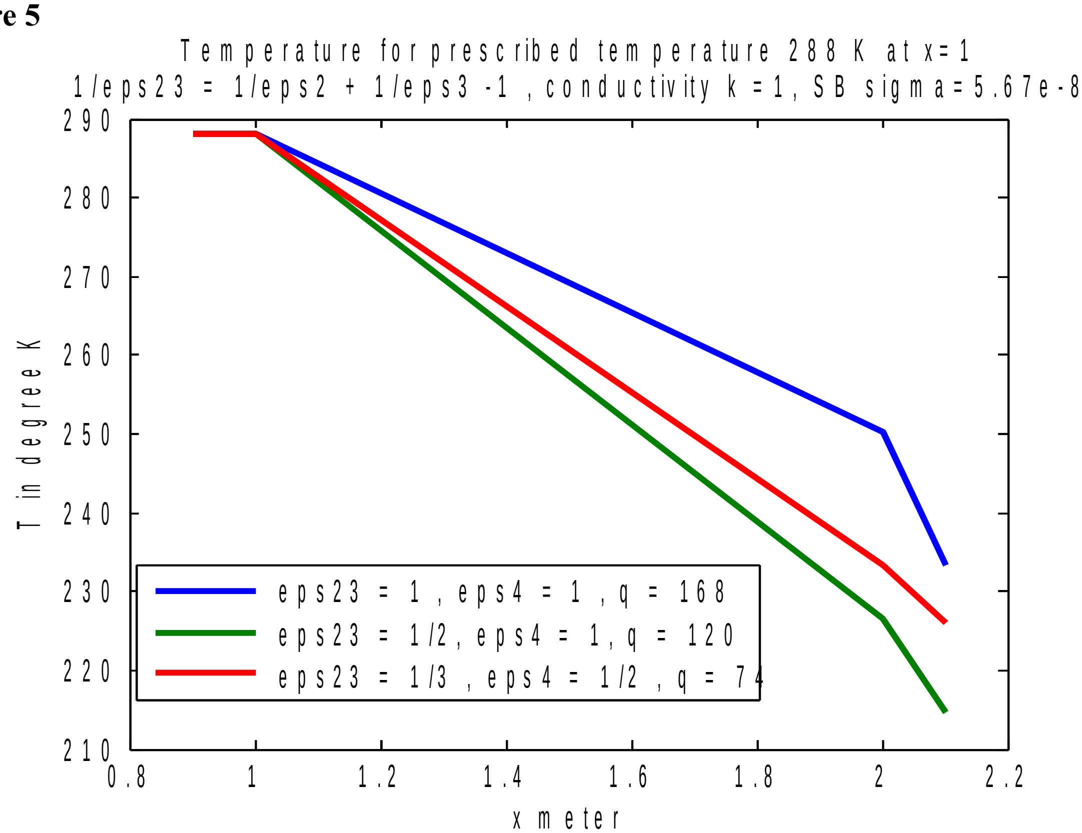


Figure 5



Conclusions

The examples with infinitely thin slabs as presented in **[1]** and in appendix **1** were already a confirmation of the conclusions of Claes Johnson: back-radiation of heat from warm to cold would be a violation of the Second Law. It does not exist.

In the examples presented in this note the infinitely thin slabs as used in **[1]** have been replaced by slabs of a finite thickness with a coefficient of heat conduction.

The one-way formulation gives continuous temperature profiles and continuous heat fluxes across the interfaces from conduction / radiation / conduction.

Two-way heat flow again has been shown not to be possible: back-radiation of heat and the huge absorption as heralded by IPCC is a fundamental error.
Universities should stop teaching the two-way formulation of heat flow by radiation, it is a crime against the Second Law of Thermodynamics.

Acknowledgment

The help of Hans Schreuder to edit this note and to host it on his site is acknowledged.

References

[1] http://www.tech-know-group.com/papers/IR-absorption_updated.pdf

[2] http://principiascientific.org/publications/PROM/PROM_REYNEN_Finite_Element.pdf

[3] http://claesjohnson.blogspot.fr

[4] http://principia-scientific.org/publications/History-of-Radiation

[5] C Christiansen, Annalen der Physik und Chemie, Leipzig,1883

[6] http://www.tech-know-group.com/essays/Greenhouse_Gases_Promote_Life.pdf

Appendix 1

One-way heat propagation formulation from the surface of the planet through an infinitely thin slab to outer space

$q \rightarrow | \rightarrow \sigma(T_1^4 - T_2^4)$ $\quad\quad$ $| \rightarrow \sigma T_2^4 = q$ $\quad$ to outer space

1 $\quad\quad$ 2

T_1 $\quad\quad$ T_2

Emission from surface 1 to slab 2: $\sigma(T_1^4 - T_2^4)$

Absorption in slab 2: $\sigma(T_1^4 - T_2^4)$

Emission from slab 2: $\sigma T_2^4 = q$

In steady state: absorption=emission: $\sigma(T_1^4 - T_2^4) = \sigma T_2^4$ (= q correct)

From which follows: $T_1^4 = 2T_2^4$

Two-way heat propagation formulation from the surface of the planet through an infinitely thin slab to outer space, giving spurious absorption.

$q \rightarrow | \longrightarrow \sigma T_1^4$ $\quad\quad$ $\sigma T_2^4 \leftarrow | \rightarrow \sigma T_2^4 = q$ $\quad$ to outer space

1 $\quad\quad$ 2

T_1 $\quad\quad$ T_2

(1) Radiation from surface 1: σT_1^4

(2) Absorption in slab 2 from surface 1: σT_1^4

(3) Back-radiation from slab2 towards surface 1: $\sigma T_2^4 = q$

(4) Radiation from slab 2 to outer space: $\sigma T_2^4 = q$

Total emission from slab2, (3)+(4): $2\sigma T_2^4 = 2q$ (impossible)

In steady state: absorption=emission : $\sigma T_1^4 = 2\sigma T_2^4 = 2q$ (impossible)

From which follows: $T_1^4 = 2T_2^4$

Conclusion of appendix 1

The temperature distribution is the same for the two formulations!
But in the two-way formulation the absorption in slab 2 is a factor 2 too big, 2q, as compared to the absorption in slab2 in the one-way formulation, the correct value q.
The two-way formulation and thereby spurious absorption and back-radiation of heat from cold to warm is wrong. This IPCC practice, discrediting CO_2 with a hypothetical huge absorption in the atmosphere and thereby back-radiation of heat is false.
Universities should stop to teach the two-way formulation for heat flow by radiation.

CO_2 is food for plants to feed the growing world population. See the paper by Carl Brehmer **[6].**

[6] http://www.tech-know-group.com/essays/Greenhouse_Gases_Promote_Life.pdf

Appendix 2

Stefan-Boltzmann relation for radiation between plates with different emission coefficients

We call $\sigma*T^4 = \theta$, it facilitates the editing.
Considering two surfaces with θ_1 and θ_2 and with emissivities ε_1 and ε_2, than we can write for a hypothetical q_1 (in the Prevost sense, from 1 in the direction of 2) and an hypothetical q_2 (from 2 in the direction of 1):

$| \longrightarrow q_1 = \varepsilon_1*\theta_1 + (1-\varepsilon_1)*q_2$ $\qquad$ $q_2 = \varepsilon_2*\theta_2 + (1-\varepsilon_2)*q_1 \longleftarrow |$

ε_1 $\qquad$ ε_2

T_1 $\qquad$ T_2

$\theta_1 = \sigma T_1^4$ $\qquad$ $\theta_2 = \sigma T_2^4$

$$q_1 = \varepsilon_1*\theta_1 + (1-\varepsilon_1)*q_2 \qquad \text{(A2.1)}$$

$$q_2 = \varepsilon_2*\theta_2 + (1-\varepsilon_2)*q_1 \qquad \text{(A2.2)}$$

The reflection $(1-\varepsilon_1)*q_2$ by surface 1 in the direction of surface 2 is taken into account, as well as the reflection $(1-\varepsilon_2)*q_1$ by surface 2 in the direction of surface 1.
We have two simultaneous linear equations for q_1 and q_2 , which can be solved analytically.

$$\begin{vmatrix} 1 & -(1-\varepsilon_1) \\ -(1-\varepsilon_2) & 1 \end{vmatrix} \begin{vmatrix} q_1 \\ q_2 \end{vmatrix} \begin{matrix} = \\ = \end{matrix} \begin{vmatrix} \varepsilon_1*\theta_1 \\ \varepsilon_2*\theta_2 \end{vmatrix} \qquad \text{(A2.3)}$$

$$\begin{vmatrix} q_1 \\ q_2 \end{vmatrix} = (1/\det) \begin{vmatrix} 1 & 1-\varepsilon_1 \\ 1-\varepsilon_2 & 1 \end{vmatrix} \begin{vmatrix} \varepsilon_1*\theta_1 \\ \varepsilon_2*\theta_2 \end{vmatrix} \qquad \text{(A2.4)}$$

$$\det = 1-(1-\varepsilon_1)*(1-\varepsilon_2) = (\varepsilon_1+\varepsilon_2-\varepsilon_1*\varepsilon_2) \qquad \text{(det stands for the determinant)}$$

For the two hypothetical fluxes q_1 and q_2 we obtain:

$$q_1 = (\varepsilon_1*\theta_1+(1-\varepsilon_1)\varepsilon_2*\theta_2)/\det \qquad \text{(A2.5)}$$

$$q2 = ((1-\varepsilon_2)\varepsilon_1*\theta_1+\varepsilon_2*\theta_2)/\det \qquad \text{(A2.6)}$$

The real heat flux from 1 to 2, for $\theta_1>\theta_2$, follows from the difference
of the two hypothetical ones:

$$q(1\rightarrow 2) = q_1-q_2 = \varepsilon_{12}*(\theta_1 - \theta_2) = \varepsilon_{12}*\sigma(T_1^4 - T_2^4) \qquad \text{(A2.7)}$$

$$1/\varepsilon_{12} = 1/\varepsilon_1 + 1/\varepsilon_2 - 1 = \det/(\varepsilon_1*\varepsilon_2) \qquad \text{(A2.8)}$$

This is the relation of Christiansen **[5]** from 1883, including reflection.
It is also given in Wikipedia, emission. For $T_1=T_2$ we find $q(1\rightarrow2) = q(2\rightarrow1) = 0$.
It reflects the second law, the heat flux is zero when the temperatures are equal.

Appendix 3

Figure 3.1

q → | slab1 | vacuum | slab2 | → σ T(4)^4 to outer space

geometry	x(1)	x(2)	x(3)	x(4)
temperatures	T(1)	T(2)	T(3)	T(4)

The conduction in slab 1 and slab 2 of figure 3.1 are modeled by 2x2 finite elements with components as function of conductivity k and the thickness:

kc1= k/(x(2)-x(1)) respectively kc2= k/(x(4)-x(3))

The 2x2 element conductivity matrices for slab 1 respectively slab 2 become:

$$\begin{vmatrix} kc1 & -kc1 \\ -kc1 & kc1 \end{vmatrix} \quad \text{respectively} \quad \begin{vmatrix} kc2 & -kc2 \\ -kc2 & kc2 \end{vmatrix}$$

The radiation process between face **2** and face **3** is modeled with a 2x2 radiation matrix with components:

kr1 = σ(T(2)^4- T(3)^4)/(T(2)- T(3))

= σ(T(2)^3 +T(2)^2*T(3) + T(2)*T(3)^2+T(3)^4)

For radiation from face 4 to outer space at node 5 with temperature zeroK=0:

kr2= σT(4)^3

The 2x2 element radiation matrices for the vacuum space and from face 4 to outer space become:

$$\begin{vmatrix} kr1 & -kr1 \\ -kr1 & kr1 \end{vmatrix} \quad \text{respectively} \quad \begin{vmatrix} kr2 & -kr2 \\ -kr2 & kr2 \end{vmatrix}$$

The matrices are assembled in the 5x5 system tri-diagonal matrix **K** relating the temperature vector **T** to the right hand side vector **rhs:**

$$\begin{vmatrix} kc1 & -kc1 & 0 & 0 & 0 \\ -kc1 & kc1+kr1 & -kr1 & 0 & 0 \\ 0 & -kr1 & kr1+kc2 & -kc2 & 0 \\ 0 & 0 & -kc2 & kc2+kr2 & -kr2 \\ 0 & 0 & 0 & -kr2 & kr2 \end{vmatrix} \begin{vmatrix} T(1) \\ T(2) \\ T(3) \\ T(4) \\ T(5) \end{vmatrix} = \begin{vmatrix} rhs(1) \\ rhs(2) \\ rhs(3) \\ rhs(4) \\ rhs(5) \end{vmatrix}$$

or

K*T=rhs

We will consider two different systems.

System1

We have as boundary condition at outer space T(5) = 0 K, and loading rhs(1)=q.
The boundary condition $T5_0$=0 is introduced by means of a Lagrangian multiplier.
For that purpose we add the unknown Lagrangian multiplier with an additional equation:

$$\left|\begin{array}{ccccc:c} & & & & & 0 \\ & & \mathbf{K} & & & 0 \\ & & 5\text{x}5 & & & 0 \\ & & & & & 0 \\ \hdashline & & & & & 1. \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right| \left|\begin{array}{c} T1 \\ T2 \\ T3 \\ T4 \\ T5 \\ \lambda \end{array}\right| = \left|\begin{array}{c} q \\ 0 \\ 0 \\ 0 \\ 0 \\ T5_0 \end{array}\right|$$

or

KL*y=rhs

$$\mathbf{y'} = \left| T1, T2, T3, T4, T5, \lambda \right|$$

$$\mathbf{rhs'} = \left| q, \ 0, \ 0, \ 0, \ 0, \ 0 \right|$$

The matrix KL can be inverted:

y = inv(KL)*rhs

The first 5 components of the vector of unknowns **y** represent the temperature vector **T,** the Lagrangian multiplier λ at position 6 is the outgoing flux to outer space.

The matrix KL depends on the temperature and an iterative solution strategy is necessary.
We start with a guess for the temperature distribution T(i).
With this distribution we calculate **y** giving rise to a new temperature distribution TN(i), from which we define dT(i)= TN(i)-T(i).
We do not update T(i) with the complete dT(i) but we use an under relaxation:

T(i)=T(i) +dT(i)/alfa with alfa = 4.

The iteration will converge to the final solution which is stopped when the mean square error in T becomes less than 0.1%.
In Appendix 4 the listing of the MATLAB program is given with green line by line comments.

System 2

In this case we have two prescribed temperatures $T2_0$ =288 K at node 2 and $T5_0$ = 0 K at outer space.
We introduce two Lagrange multipliers $\lambda 1$ for the boundary condition in node 2 and $\lambda 2$ for the boundary condition in outer space at node 5.
They have position 6 respectively 7 in the **rhs** vector.

$$
\left|\begin{array}{ccccc:cc}
 & & & & & 0 & 0 \\
 & & \mathbf{K} & & & 1 & 0 \\
 & & 5\text{x}5 & & & 0 & 0 \\
 & & & & & 0 & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0
\end{array}\right|
\left|\begin{array}{c} T1 \\ T2 \\ T3 \\ T4 \\ T5 \\ \lambda 1 \\ \lambda 2 \end{array}\right|
=
\left|\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ T2_0 \\ T5_0 \end{array}\right|
$$

or

KL*y=rhs

y = inv(KL)*rhs

y'= $|T1, T2, T3, T4, T5, \lambda 1, \lambda 2|$

rhs' = $|\ 0,\ 0,\ 0,\ 0,\ 0, 288, 0\ |$

The Lagrange multipliers with position 6 respectively 7 in the vector of unknowns represent the incoming flux at node 2 respectively the outgoing flux to outer space at node 5.
In appendix 4 the listing of the Matlab program is given with line by line green comments.

```
%engineeringproof.m

x=[0.9,1,2,2.1]                     %geometry, meters
T=ones(4,1);                        %vector of temperatures K
q=240                               %prescribed flux at x(1)
T=288*T;                            %guess of temperatures
T(2)=T(1)-40;
T(3)=T(2)-40;
T(4)=T(3)-40;

sigma=5.67e-8;                     %Stefan-Boltmann constant W/m^2/K^4
eps2=[1,0.5];
eps3=[1,0.5];
eps4=[1,0.5];
kc=1;                         %conductivity taken as 1 W/m/K
kc12=kc/(x(2)-x(1));          %component of conductivity matrix between
                                  %node 1 and 2
kc34=kc/(x(4)-x(3));         %component of conductivity matrix between
                                 %node 3 and 4
for j=1:2                        % j=1 prescribed heat flux at node 1
 figure                                % j=2 prescribed temperature at node 2
  for k=1:3
   if k==1                       %k=1 eps=1 at face 2,3 and 4
     eps23=eps2(1);              %eps23 =1
     eps45=eps4(1);              %eps45=1
   end
   if k==2                      %k=2  eps2=0.5 eps3=1 eps4=1
       eps23= 1/(1/eps2(2)+1/eps3(1)-1);   %eps23=0.5
       eps45=eps4(1);                      %eps45=1
   end
   if k==3                       %k=3  eps2=1 eps3=0.5 eps4=0.5
       eps23= 1/(1/eps2(2)+1/eps3(2)-1); %eps23 = 0.3333
       eps45=eps4(2);                    %eps45 = 0.5

   end
   sigma23=eps23*sigma;
   sigma45=eps45*sigma;


error=1;
while error>0.001     %system matrix to calculate temperatures depend on
                      % temperature because of radiation which is not linear
                      %we have an iterative solution until error < 0.1%
    K=zeros(5);       % refresh system matrix

kr23=sigma23*(T(2)^3+T(2)^2*T(3)+T(2)*T(3)^2+T(3)^3);
                     %component of radiation matix between nodes 2 and 3
kr45=sigma45*T(4)^3;
                     %component of radiation matix between nodes 4 and 5
                     %node 5 is outerspace at zeroK
%assembling of system matrix
K(1,1)=K(1,1)+kc12;
K(1,2)=K(1,2)-kc12;
K(2,1)=K(2,1)-kc12;
K(2,2)=K(2,2)+kc12;
K(2,2)=K(2,2)+kr23;
K(2,3)=K(2,3)-kr23;
K(3,2)=K(3,2)-kr23;
K(3,3)=K(3,3)+kr23;
K(3,3)=K(3,3)+kc34;
```

```
K(3,4)=K(3,4)-kc34;
K(4,3)=K(4,3)-kc34;
K(4,4)=K(4,4)+kc34;
K(4,4)=K(4,4)+kr45;
K(4,5)=K(4,5)-kr45;
K(5,4)=K(5,4)-kr45;
K(5,5)=K(5,5)+kr45;
if j==1               %j=1 node 5 is zeroK outerspace, introduced by
                      %Lagrange multipliers
LAG=[0,0,0,0,1];      %condition matrix according to Lagrange
KL=[K,LAG';LAG,0];    %system matrix modified by Lagrange multipliers for BC
rhs =[q,0,0,0,0,0];   %rhs(1) at node 1 = q


else                          %j=2 prescrbed temperature at node 2 and
                              % node 5 = outerspace

LAG=[0,1,0,0,0;0,0,0,0,1];    %prescribed temperature at node 2
                              %zeroK at node 5 = outerspace
KL=[K,LAG';LAG,zeros(2,2)]; %system matrix with boundary conditions
                             %by means of Lagrange multipliers
rhs = [0,0,0,0,0,288,0];    %rhs(5) prescribed temperature 288 at position 5
                            %corresponding to lagrange multiplier for
                            % temperature at node 2
                            %rhs(6)prescribed temperature zeroK at position 6
                            %corresponding to lagrange multiplier for
                            % temperature at node 2



end
invKL=inv(KL);              %invert systemmatrix KL with boundary condition

y=invKL*rhs';               % y is vector of 5 temperatures and for j=1 one
                            %Lagrange Multiplier and for j=2 two Multipliers
for i=1:4                  % put the first 5 components of y in temperature
vector
                            % and compare with earlier results from the
                            % previous iteration step.

    dT(i)=y(i)-T(i);
    T(i)= T(i)+dT(i)/4;  %use under relaxation to approach the solution
end
error=sqrt(dT*dT'/4);    % dT*dT'/4 = (dT(1)^2+dT(2)^2+dT(3)^2+dT(4)^2)/4 =
                         %             average absolute error
end

%iteration has converged
Tv(k,:)=T;       %store temperature in a matrix for each k
%print output
k
j
eps23
eps45
   xtitle2=['1/eps23 = 1/eps2 + 1/eps3 -1 , conductivity k =1, SB sigma=5.67e-
8'];
if j==1
    xtitle1= ['Temperature for heat input q=240 W/m^2 at x=0.9']

    qin=q
    qout=-y(6)
```

```
end
if j==2

    xtitle1= ['Temperature for prescribed temperature 288 K at x=1']
    qin= -y(6)
    qout=-y(7)
end
Ttransp=T'

  if k==1
      eps23
      plot(x,T,'linewidth',2)
      title({xtitle1})
  xlabel('x meter')
  ylabel('T in degree K')
  figure
  end

  end
  plot(x,Tv,'linewidth',2)
  title({xtitle1;xtitle2})
  xlabel('x meter')
  ylabel('T in degree K')
  if j==1
  legend('eps23 = 1 , eps4 = 1',...
         ' eps23 = 1/2, eps4 = 1 ',...
         'eps23 = 1/3 , eps4 = 1/2 ,')
  else
  legend(' eps23 = 1 , eps4 = 1 , q = 168',...
         ' eps23 = 1/2, eps4 = 1, q = 120 ',...
         ' eps23 = 1/3 , eps4 = 1/2 , q = 74')
  end
end
```